Casimir effect in Napoli

Giuseppe Bimonte
Physics Department
Università di Napoli Federico II-ITALY
INFN- Sezione di Napoli

Overview

- Our group and collaborations
- Our interests in Casimir physics
- Perspectives and conclusions

Our group and collaborations

Our group	• Giuseppe Bimonte (Univ. Federico II Napoli & INFN) • Enrico Calloni (Univ. Federico II Napoli) • Giampiero Esposito (INFN) • Luigi Rosa (Univ. Federico II Napoli & INFN)
The Aladin collaboration (more from Luigi's talk)	• Detlef Born (Univ. Federico II Napoli) • F. Gatto (INFN – Sez. Genova) • Uwe Huebner (Inst. Photonic Tech. Jena) • Evgeni Il'ichev (Inst. Photonic Tech. Jena) • Francesco Tafuri (Seconda Univ. Napoli) • R. Vaglio (Univ. Federico II Napoli)
Padova collaboration	• Giovanni Carugno (INFN - Sez. Padova) • Giuseppe Ruoso (INFN - Lab. Legnaro) • Piergiorgio Antonini (INFN - Padova)
Leipzig collaboration	• Vladimir Mostepanenko (Leipzig Univ.) • Galina Klimchitskaya (Leipzig Univ.)

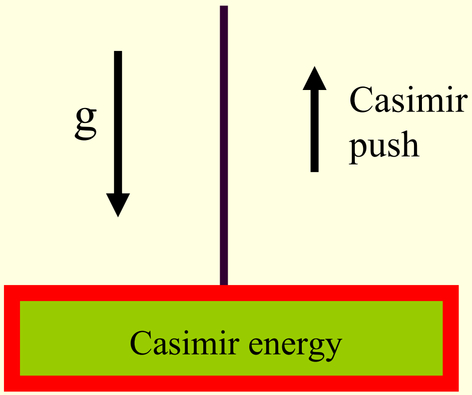
Our interests in Casimir physics

- Casimir effect in a gravitational field
- Casimir effect in superconductors
- Thermal features of the Casimir effect

Casimir effect in a gravitational field (or the weight of the vacuum)

We studied the possibility of testing experimentally if the Equivalence Principle of General Relativity holds for vacuum fluctuations:

"What is the weight of Casimir energy?"



Explicit evaluation of the vev for the Maxwell stress tensor in a weak gravitational field gives expected result for the Casimir push W_C:

$$W_C = M_C\, g \qquad M_c = E_c/c^2$$

Since the Casimir energy E_c of a plane parallel cavity is negative, it contributes a negative weight!

The Casimir push is very very small. For a cavity with an area of 1 cm^2 and a plate separation of 100 nm:

$$M_c = -\,4.8\; 10^{-28}\ \mathrm{Kg}$$ (less than a proton mass in magnitude)

Detection is not hopeless though if one can find means of modulating the force in time

M.T. Jaekel and S. Reynaud, J. Phys. I (France) 3, 1093 (1993) (on the inertia of Casimir energy in 2 dimensions)
E. Calloni, L. Di Fiore, G. Esposito, L. Milano and L. Rosa, Phys. Lett. A297 (2002) 328;
G. Bimonte, E. Calloni, G. Esposito and L. Rosa, Phys.Rev. D74 (2006) 085011 (with Errata); D76 (2007) 025008;
G. Bimonte, G. Esposito and L. Rosa, Phys. Rev. D78 (2008) 024010.
S.A. Fulling, K.A.Milton, P.Parashar, A.Romeo,K.V. Shajesh, J Wagner Phys. Rev.D76 (2007), 025004.
G. Esposito, G. Napolitano and L. Rosa, Phys. Rev. D77 (2008) 105011; D78 (2008) 107701.

Casimir effect in superconductors

- The ALADIN experiment aims at probing how the superconducting phase transition affects Casimir energy (details in Luigi's talk).

G. Bimonte, E. Calloni, G. Esposito, L. Milano and L. Rosa, Phys. Rev. Lett.94 (2005) 180402

G. Bimonte, E. Calloni, G. Esposito and L. Rosa, Nucl. Phys. B726 (2005) 441

- We recently proposed to use a superconducting Casimir apparatus to probe controversial features of the thermal Casimir effect in metallic systems

G. Bimonte, arXiv:0807.2950 (in press on Phys. Rev. A)

Thermal Casimir effect in metallic systems

For an ideal cavity with perfectly reflecting mirrors (Casimir 1948):

$$F(a) = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}$$

Modern experiments require considering a number of corrections:

- Surface roughness and shape of plates
- Finite conductivity of the plates
- Temperature of the plates

Surprisingly, **for metallic plates**, **combined** effect of temperature and finite conductivity raises severe problems.

Thermal Casimir effect for metals

The combined effect of finite conductivity and temperature, as given by Lifshitz theory, **strongly** depends on the model used for metal.

M. Bostrom and B.E. Sernelius, Phys. Rev. Lett.84 (2000) 4757

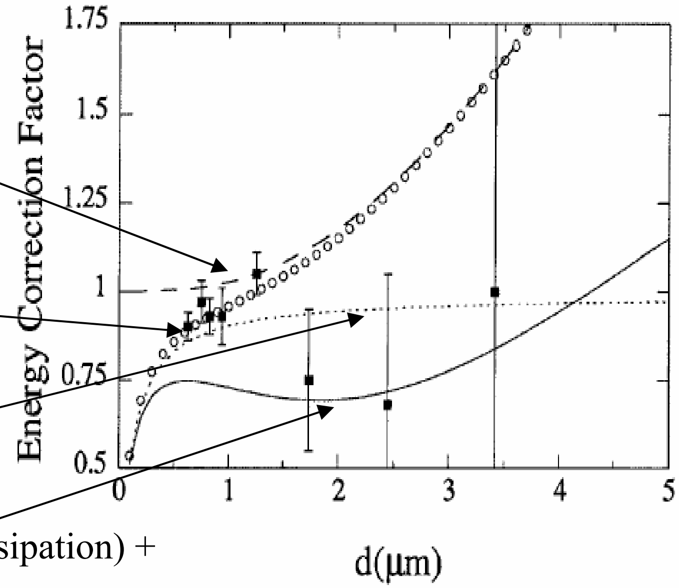
Energy correction factor η_E $\eta_E = E/E_{id}$

Picture from Bostrom and Sernelius PRL 84 (2000) 4757

Thermal correction for ideal plates

Thermal correction plasma model

Plasma model, zero temperature



Drude model (with dissipation) + temperature

FIG. 1. The energy correction factor for Au at 0 K (dotted line), Au at 300 K (solid line), Au at 300 K with the static transverse electric part incorrectly treated as in the perfect conductor case (circles), and finally the energy between perfect conductors evaluated at 300 K (dashed line). We have, as a comparison, also plotted the experimental energy of Ref. [5] (squares).

Dissipation produces a large **repulsive** force $\Delta P(T)$

$$\Delta P(T) \cong \frac{k_B T}{8\pi a^3}\zeta(3)\left(1 - 6\frac{\delta}{a} + 24\frac{\delta^2}{a^2}\right)$$

where $\delta = c/\Omega_p$ with Ω_p plasma frequency

Where does this repulsive force come from?

Experimental points from Lamoreaux, PRL 78 (1997) 5; 81 (1998) 5475.

A deeper physical insight is achieved by **separating** the **thermal correction** from the contribution of zero-point fluctuations, and by looking at the spectrum along the **real-frequency** axis

J. R. Torgerson and S. Lamoreaux, Phys. Rev. E70 (2004) 047102;
G. Bimonte, Phys. Rev. E73 (2006) 048101

Split the Casimir Pressure P(a,T) as

$$P(a,T) = P_0(a,T) + \Delta P(a,T)$$

Contribution of zero-point fluctuations ($P_0(a,T)$)

Contribution of Thermal radiation ($\Delta P(a,T)$)

Different frequencies contribute to $P_0(a,T)$ and $\Delta P(a,T)$

$$P_0(a,T) = -\frac{\hbar}{2\pi^2}\int_0^\infty d\omega \int_0^\infty d\,k_\perp k_\perp \,\mathrm{Re}\left\{k_z \sum_{\alpha=\mathrm{TE,TM}}\left[1-\frac{e^{-2\,i\,k_z\,a}}{r_\alpha^{(1)} r_\alpha^{(2)}}\right]^{-1}\right\}$$

$P_0(a,T)$ depends on T as a parameter and receives contributions form frequencies around $\omega_c = c/2a$. Dissipation has little effect on $P_0(a,T)$

$$\Delta P(a,T) = -\frac{\hbar}{\pi^2}\int_0^\infty d\omega \int_0^\infty d\,k_\perp k_\perp \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right)-1}\mathrm{Re}\left\{k_z \sum_{\alpha=\mathrm{TE,TM}}\left[1-\frac{e^{-2\,i\,k_z\,a}}{r_\alpha^{(1)} r_\alpha^{(2)}}\right]^{-1}\right\}$$

$\Delta P(a,T)$ depends on low frequencies from $k_B T/\hbar$ (infrared) down to microwaves.

Spectrum of the thermal correction

J. R. Torgerson and S. Lamoreaux, Phys. Rev. E70 (2004) 047102;
G. Bimonte, Phys. Rev. E73 (2006) 048101

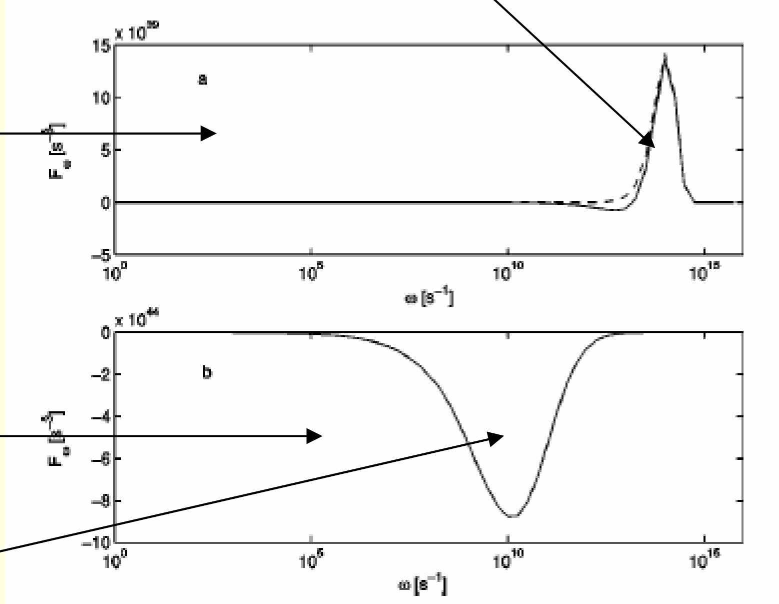
The large thermal correction arises from thermal evanescent waves with transverse electric polarization (TE EW)

Results for the Drude-model

$$\omega_T = \frac{k_B T}{\hbar} = 3.9 \times 10^{13}\ \mathrm{rad/s}$$

Contribution to ΔP(a,T) from
TE Propagating waves
Dashed-line is for ideal metal

Contribution to ΔP(a,T) from
TE Evanescent waves (TE EW)
For zero dissipation this is zero.



$$\tilde{\omega} = \gamma \left(\frac{\omega_c}{\Omega_P}\right)^2 = 6.4 \times 10^9\ \mathrm{rad/s}$$

$$\omega_c = \frac{c}{2a}$$ characteristic frequency of the cavity

Solid line is for Au with T=300 K, a=1 μm.
Dashed-line is for ideal metal.
(from Torgerson and Lamoreaux, PRE 70 (2004) 047102)

The repulsive contribution from thermal EW

J. R. Torgerson and S. Lamoreaux, Phys. Rev. E70 (2004) 047102;
G. Bimonte, Phys. Rev. E73 (2006) 048101

$$\Delta P_{\rm EW}(a,T) = \frac{\hbar}{\pi^2}\int_0^\infty d\omega \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right)-1}\int_0^\infty dq\, q^2 \times \sum_{\alpha={\rm TE,TM}} {\rm Im}\left[1-\frac{e^{2qa}}{r_\alpha^{(1)}(\omega,k_\perp)r_\alpha^{(2)}(\omega,k_\perp)}\right]^{-1}$$

Distinctive features of $\Delta P_{EW}(a,T)$ are

- It is always **repulsive**
- It does not vanish in the limit of vanishing dissipation
- It is **zero for strictly zero dissipation** (plasma model)
- For metals without impurities, it violates Nernst heat theorem

VB Bezerra, G.L. Klimchitskaya and VM Mostepanenko.Phys. Rev. A 65 (2002) 052113;
I. Brevik, SE Ellingsen and KA Milton, New J. Phys. 8 (2006) 236

- It involves low frequencies $\omega \approx \gamma\,(\Omega_c/\Omega_p)^2$, with $\Omega_c = c/(2a)$

Remark: for vanishing dissipation, the plasma model results are recovered smoothly in the propagating sector and in the TM evanescent sector.

Summary of the Physics of the problem

Ideal mirrors at T=0: cavity has standing modes → TM n=0,1,2,3... / TE n= 1,2...

Casimir Energy ⟺ Zero-point energy of cavity modes

Ideal mirrors at T>0: cavity modes get populated

Free Energy receives contribution from thermal photons

Finite plasma frequency (any T): cavity modes penetrate walls a bit (finite skin depth).

} **NO TE EW**

Dissipation:
- spectrum of modes broadens a bit (small thermal correction)
- **TE EW appear** ⇒ **Large thermal correction**

What physical effect causes the large thermal TE EW contribution?

G. Bimonte, New. J. Phys. 9, 281 (2007)

The **sudden** appearence of a new sector of e.m. fluctuations as soon as dissipation is turned on indicates that these fluctuations are related to a **NEW physical phenomenon characteristic of conductors**, that is absent when dissipation is zero.

Hint: the relevant low-frequency thermal TE EW consist of a continuous spectrum of

fluctuating magnetic fields.

What produces these fields?

The Johnson-Nyquist currents (1928) inside the plates.

↓

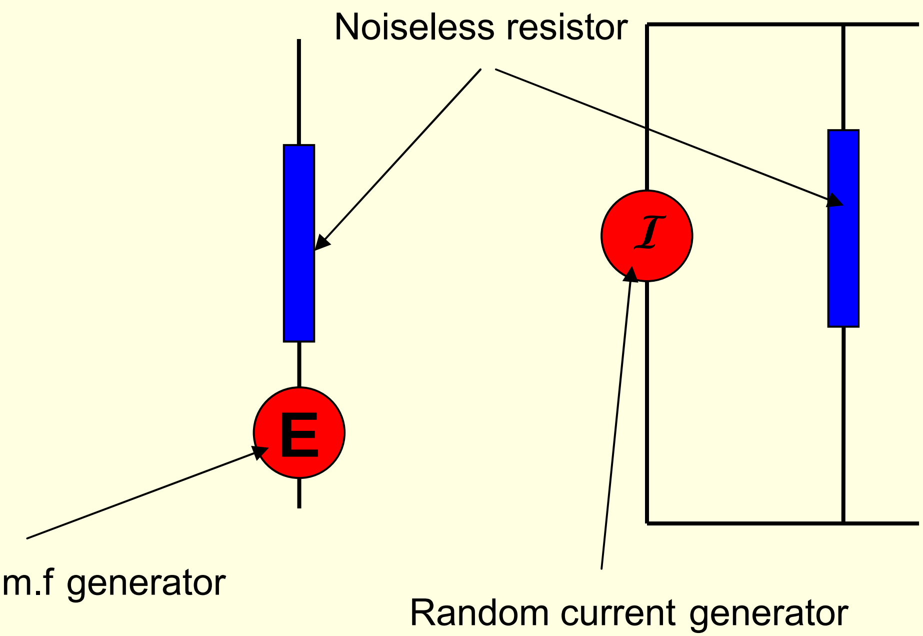
Physical Picture: Johnson currents in either plate induce (correlated) eddy currents in the other plate. Repulsion arises from the magnetic interaction between them.

Johnson noise

Johnson noise is the electronic noise generated by thermal agitation of charge carriers inside a conductor at thermal equilibrium

G. Bimonte, New. J. Phys. 9, 281 (2007)

a noisy resistor can be modelled by a noiseless resistor connected to either a e.m.f. or a current noise generator



$$\langle E(\omega) E^*(\omega') \rangle = 4\pi R \frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} \delta(\omega - \omega')$$

$$\langle I(\omega) I^*(\omega') \rangle = 4\pi \frac{1}{R} \frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} \delta(\omega - \omega')$$

Nyquist (1928)

$$\frac{\hbar\omega}{k_B T} \ll 1$$

$$\langle E(\omega) E^*(\omega') \rangle = 4\pi R\, k_B T \, \delta(\omega - \omega')$$

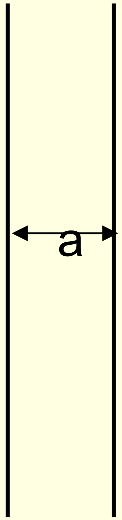
$$\langle I(\omega) I^*(\omega') \rangle = 4\pi \frac{1}{R} k_B T \, \delta(\omega - \omega')$$

White noise spectrum

Thermal interaction between two nearby metallic wires

G. Bimonte, New. J. Phys. 9, 281 (2007)

Two nearby metallic wires



The circuit equations (low-frequency approx.)

$$\mathcal{L}\,\frac{di_1}{dt} + \mathcal{M}(\vec{a})\frac{di_2}{dt} + R\,i_1 = \mathcal{E}_1(t)$$

$$\mathcal{L}\,\frac{di_2}{dt} + \mathcal{M}(\vec{a})\frac{di_1}{dt} + R\,i_2 = \mathcal{E}_2(t)$$

Wires self-inductance

Random e.m.f.

Wires mutual inductance

$$\langle \mathcal{E}_i(\omega)\,\mathcal{E}_j^*(\omega')\rangle = 4\pi R\,\frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_B\,T}\right)-1}\,\delta(\omega-\omega')\,\delta_{i\,j}$$

Force between the wires: $\vec{F}_{12}(\vec{a}) = \langle i_1\, i_2\rangle\,\vec{\nabla}_a\mathcal{M}(\vec{a})$

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

$$\vec{F}_{12} = -k_B T \, H \, \vec{\nabla}_a (m^2) \qquad m = \mathcal{M}/\mathcal{L} \qquad \omega_R = R/\mathcal{L}$$

$$H = \frac{1}{\pi} \int_0^\infty d\omega \, \omega E\left(\frac{\omega}{\omega_T}\right) \mathrm{Im}\left[(\omega_R - i\omega)^2 + \omega^2 m^2\right]^{-1} \qquad E(y) = \frac{y}{e^y - 1}$$

The force is always repulsive!

For vanishing R, the force **does not** vanish

$$\lim_{R \to 0} \vec{F}_{12} = -k_B T \, f(m^2) \, \vec{\nabla}_a (m^2)$$

Free-energy

$$\vec{F}_{12} = -\vec{\nabla}_a \mathcal{F}$$

$$\mathcal{F} = \frac{k_B T}{\pi} \int_0^\infty \frac{d\omega}{\omega} E\left(\frac{\omega}{\omega_T}\right) \mathrm{Im} \log\left[1 + \left(\frac{\omega \, m}{\omega_R - i\,\omega}\right)^2\right]$$

For low T (in metals without impurities) R vanishes like T^2. Then

$$\mathcal{F} \approx g(m^2) \, k_B T \quad \Rightarrow \quad \lim_{T \to 0} S = -k_B \, g(m^2) \equiv S_0 < 0.$$

Nernst th. violated

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

Summarizing: the electrodynamic interaction between Johnson and eddy currents gives rise to a force that:

- is repulsive
- does not vanish for $R \to 0$
- vanishes for strictly dissipationless wires $R=0$
- violates Nernst th. (for wires with no impurities)

All this is as in the thermal Casimir effect

supporting our physical picture that the large thermal correction to the Casimir pressure, found from Lifshitz theory when dissipation is taken into account, **originates from Johnson currents in the plates.**

Interestingly, in the wires case, thermodynamical inconsistencies at zero temperature can be resolved by taking account of the capacitances of the wires (edge effect) (see G. Bimonte, New. J. Phys. 9, 281 (2007))

A big puzzle

Proximity effects of Johnson noise above one μm are observed in several experiments with ultracold atoms in magnetic traps, indicating good agreement with theory

MPA Jones, CJ Vale, D Sahagun, BV Hall and EA Hinds, Phys. Rev. Lett. 91 (2003) 080401;
YJ Lin, I Teper, C Chin and V Vuletic, Phys. Rev. Lett. 92 (2004) 050404;
B Zhang, C Henkel, S Wildermuth, S Hofferberth, P Kruger and J Schmiedmayer Eur. Phys. J.D35 (2005) 97;
DM Harber, JM McGuirk, JM Obrecht and EA Cornell, J. Low Temp. Phys. 133 (2003) 229;
C. Henkel, S Potting and M Wilkens, Appl. Phys. B69 (1999) 379

However, recent Casimir experiments at Purdue University, using micromachined torsional oscillators, seem to exlude the repulsive thermal force arising from Johnson noise in the plates.

R.S. Decca et al., Ann. Phys. 318 (2005) 318; Phys. Rev. D75, (2007) 077101; Eur. Phys. J. C51 (2007) 963

Theoretical conundrum: what suppresses the repulsive effect of Johnson noise in Casimir experiments?

Question: are there other means to probe proximity effects related to Johnson noise, at submicron scales?

How to probe proximity effects of Johnson noise at submicron separation?

We proposed two possible experimental approaches

- Radiative heat transfer between closely spaced metallic bodies

  G. Bimonte, Phys. Rev. Lett 96 (2006) 160401;
  V.B. Bezerra, G. Bimonte, G.L. Klimchitskaya, V.M. Mostepanenko and C. Romero, Eur.Phys. J. C52 (2007) 701;

- Measurement of temperature dependence of the Casimir force in superconducting systems

  G. Bimonte, arXiv:0807.2950 (in press on Phys. Rev. A)

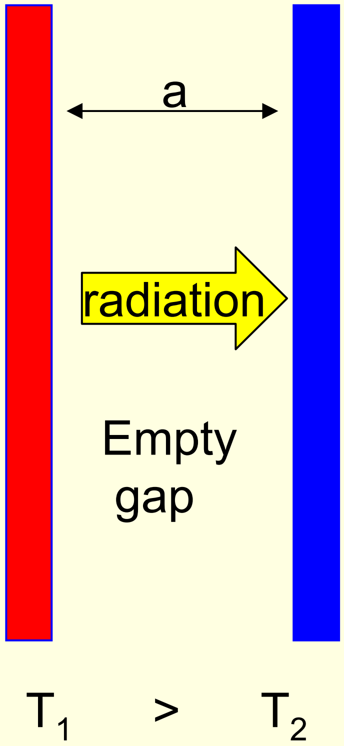
TE EW and heat transfer

Thermal EW associated with Johnson noise give the dominant contribution to radiative heat transfer between two metallic surfaces, separated by an empty gap, at submicron separations

D Polder and M.Van Hove, Phys. Rev. B4 (1971) 3303.

The frequencies involved are same as in thermal corrections to the Casimit force. Therefore, heat transfer gives information on thermal TE EW

G. Bimonte, Phys. Rev. Lett 96 (2006) 160401.



Near field radiative heat transfer

The contribution to S from EW :

$$S_{\rm EW} = \frac{\hbar}{\pi^2}\int_0^\infty d\omega\,\omega\left(\frac{1}{\exp(\hbar\omega/k_B T_1)-1} - \frac{1}{\exp(\hbar\omega/k_B T_2)-1}\right)\int_0^\infty dq\,q$$
$$\times \sum_{\alpha={\rm TE,TM}} \frac{{\rm Im} r_\alpha^{(1)}(\omega,k_\perp)\,{\rm Im} r_\alpha^{(2)}(\omega,k_\perp)}{|1-r_\alpha^{(1)}(\omega,k_\perp) r_\alpha^{(2)}(\omega,k_\perp)\exp(-2\,q\,a)|^2}\, e^{-2qa}\,. \quad ($$

Note again that S_{EW} vanishes if the reflection coefficients are real

We have compared the powers S of heat transfer impled by various models of dielectric functions and surface impedances, that are used to estimate the thermal Casimir force

V.B. Bezerra, G. Bimonte, G.L. Klimchitskaya, V.M. Mostepanenko and C. Romero, Eur.Phys. J. C52 (2007) 701.

Models for the metal

V.B. Bezerra, G. Bimonte, G.L. Klimchitskaya, V.M. Mostepanenko and C. Romero, Eur.Phys. J. C52 (2007) 701.

- The Drude model (Lifshitz theory):

$$\varepsilon_D = 1 - \frac{\Omega^2}{\omega(\omega + i\gamma)}$$

- The surface impedance of the normal skin effect Z_N:

$$Z_N = (1-i)\sqrt{\frac{\omega}{8\pi\sigma_0}}$$

- The surface impedance of the Drude model Z_D:

$$Z_D = \frac{1}{\sqrt{\varepsilon_D}}$$

- A modified form of the surface impedance of infrared optics, including relaxation effects Z_t

$$Z_t = -i\frac{\omega}{\sqrt{\omega_p^2 - \omega^2}} + Z_t'$$

Comparison of powers S of heat transfer

V.B. Bezerra, G. Bimonte, G.L. Klimchitskaya, V.M. Mostepanenko and C. Romero, Eur.Phys. J. C52 (2007) 701.

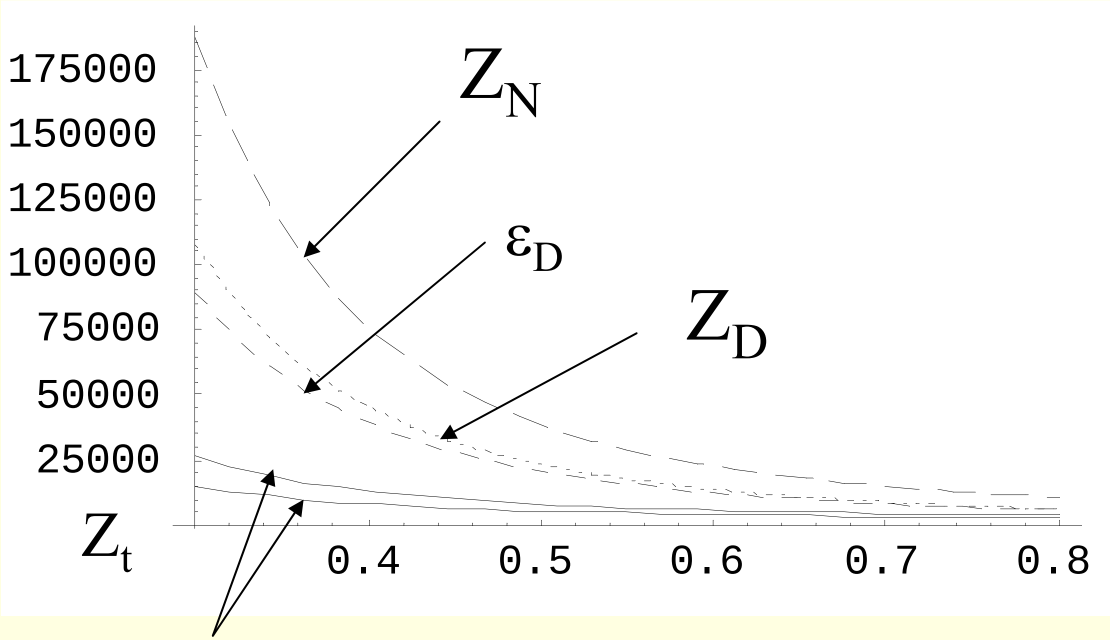
$$\varepsilon_D = 1 - \frac{\Omega^2}{\omega(\omega + i\gamma)}$$ (Lifshitz theory)

$$Z_D = \frac{1}{\sqrt{\varepsilon_D}}$$

$$Z_N = (1-i)\sqrt{\frac{\omega}{8\pi\sigma_0}}$$

Z_t optical data + extrapolation to low frequencies

S (in erg cm^{-2}sec^{-1}))



separation in μm

Experiments on near field radiative heat transfer

Few experiments exist to probe near field enhancement of heat transfer

- Hargreaves measured S for two chromium plates, for gaps up to 1 mm demonstrating enhancement. But total power still less than 50% of blackbody radiation

  C.M. Hargreaves, Philips Res. Rep., Suppl. 5 (1973) 1.

- Recently, Chen Gang and collaborators, at MIT, measured S for a silica sphere and a glass flat substrate, observing strong near field effects, over blackbody level. An experiment with metallic bodies is planned, to test features relevant for the thermal Casimir effect.

  A. Narayanaswamy, S. Shen and G. Chen, Phys. Rev. B78 (2008) 115303.

Casimir effect in superconductors

G. Bimonte, arXiv:0807.2950 (in press on Phys. Rev. A)

We consider a Casimir apparatus consisting of either two superconducting plates (Nb-Nb) or one superconducting and one normal plate (Nb-Au). The prescription for the TE zero mode greatly affects how the Casimir force changes when the temperature T is changed

The plasma prescription (i.e. no dissipation) implies a temperature independent reflection coefficient for the TE zero mode, and so it predicts an unmeasurably small change $\Delta P|_{pl}$ of Casimir force when temperature is varied.

Since the fraction n_s/n_n of superconducting electrons depends on T, the Drude prescription implies a temperature-dependent reflection coefficient for the TE zero mode, and so it predicts a non-zero change $\Delta P|_{Dr}$ of Casimir force when T is varied

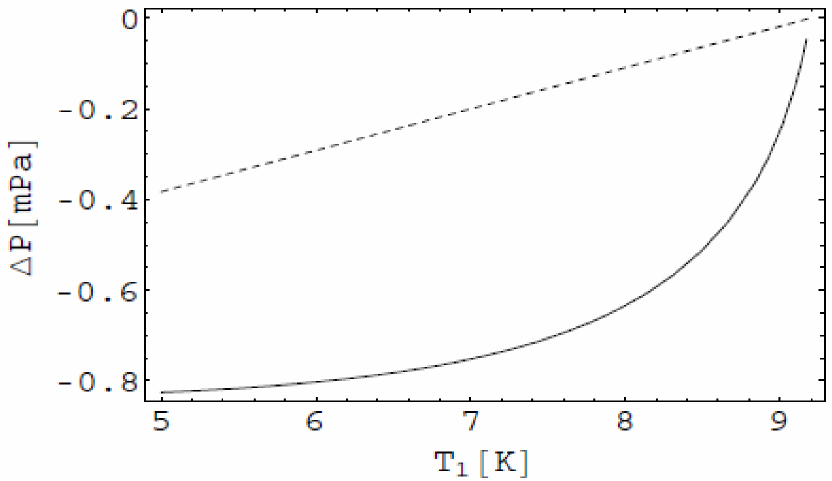


FIG. 1: Plots of $\Delta P(\mathrm{Nb}-\mathrm{Nb})|_{\mathrm{Dr}}$ (solid line) and $\Delta P(\mathrm{Nb}-\mathrm{Au})|_{\mathrm{Dr}}$ (dashed line) (in mPa) for two parallel plates at fixed separation $a = 150$ nm, versus temperature T_1 (in K), for fixed $T_2 \simeq T_c$.

Conclusions

- The thermal Casimir effect poses puzzling problems
- Johnson noise provides the physical explanation of the large thermal correction to the Casimir pressure in metallic cavities, when dissipation is considered
- Johnson noise exists only in conductors with ohmic dissipation. It is therefore explained why large thermal corrections are absent in models which neglect dissipation
- The problem can be clarified by investigating experimentally other proximity phenomena invoving Johnson noise, like radiative heat transfer and the temperature dependence of the Casimir force in superconducting devices

Mathematical origin of the large thermal correction in metals

M. Bostrom and B.E. Sernelius, Phys. Rev. Lett.84 (2000) 4757

Using complex integration techniques, one can write the free energy as a sum over discrete imaginary frequencies ξ_l (Matsubara frequencies):

$$F_E(a,T)=\frac{k_BT}{4\pi}\sum_{l=-\infty}^{\infty}\int_0^{\infty}k_\perp dk_\perp\{\ln[1-r_{||}^2(\xi_l,k_\perp)e^{-2aq_l}]+\ln[1-r_\perp^2(\xi_l,k_\perp)e^{-2aq_l}]\},$$

$$\xi_l=2\pi lk_BT/\hbar$$

$$l=\ldots,-2,-1,0,1,2,\ldots$$

Zero-mode

$$r_{||}^2(\xi_l,k_\perp)=\left[\frac{\varepsilon(i\xi_l)q_l-k_l}{\varepsilon(i\xi_l)q_l+k_l}\right]^2,\qquad r_\perp^2(\xi_l,k_\perp)=\left(\frac{q_l-k_l}{q_l+k_l}\right)^2$$

TM TE

$$q_l=(\xi_l^2/c^2+k_\perp^2)^{1/2},\quad k_l=[\varepsilon(i\xi_l)\xi_l^2/c^2+k_\perp^2]^{1/2}$$

Drude model prescription

$$\varepsilon_D(i\xi)=1+\frac{\omega_p^2}{\xi(\xi+\gamma)}.$$

Dissipation

$\Rightarrow$ $r_{||}^2(0,k_\perp)=1,\ r_\perp^2(0,k_\perp)=0.$ $\Rightarrow$ TE zero-mode gives zero

Plasma model prescription

$$\varepsilon_p(i\xi)=1+\frac{\omega_p^2}{\xi^2}.$$

$\Rightarrow$ $r_{||}^2(0,k_\perp)=1,\ r_\perp^2(0,k_\perp)=\left(\frac{ck_\perp-\sqrt{\omega_p^2+c^2k_\perp^2}}{ck_\perp+\sqrt{\omega_p^2+c^2k_\perp^2}}\right)^2.$ $\Rightarrow$ TE zero-mode gives contribution

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

Question: have we missed anything?

Yes: a **finite** wire has end-points, where **charges build up**

In our circuit equations we should take account of this, by including a **capacitance**

$$\mathcal{L}\,\frac{di_1}{dt} + \mathcal{M}(\vec{a})\frac{di_2}{dt} + R\,i_1 + \frac{Q_1}{C} = \mathcal{E}_1(t)$$
$$\mathcal{L}\,\frac{di_2}{dt} + \mathcal{M}(\vec{a})\frac{di_1}{dt} + R\,i_2 + \frac{Q_2}{C} = \mathcal{E}_2(t)$$

Capacitances act as **high-pass filters** and block low frequencies

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

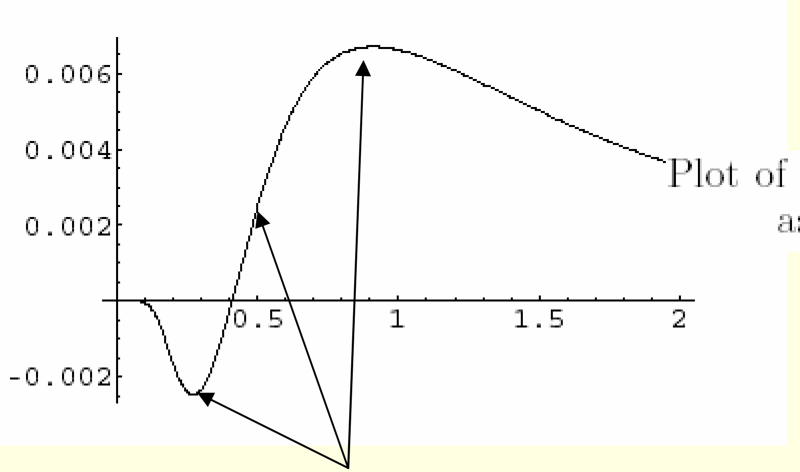
Inclusion of capacitances ensures that

- the force vanishes for vanishing R
- the entropy vanishes for T=0

Indeed

$$\mathcal{F} = -\frac{16\pi^5\, m^2}{63} \left(\frac{k_B T}{\hbar\,\omega_C}\right)^6 \hbar\omega_R \, .$$

$$\omega_C = 1/\sqrt{\mathcal{L}\, C}.$$



Plot of the free energy (in units of $\hbar\omega_C$) as a function of $t = k_B T/(\hbar\omega_C)$

There still is a range of T for which Entropy is negative

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

However: each wire being a RLC circuit, it possesses an associated free energy

$$\mathcal{F}_{\rm self} = k_B T \, \log[1 - \exp(-\hbar\omega_C/(k_B T))]$$

Inclusion of the wires self-entropy makes the **total** entropy of the system **positive** at all temperatures, while respecting Nernst th.

Inclusion of capacitances for the end-points gives a fully satisfactory picture

Thermal interaction between two nearby noisy resistors

G. Bimonte, New. J. Phys. 9, 281 (2007)

Are edge effects important in the case of bulk plates of size L?

It depends

Recall that typical size of current fluctuations is of order of plates separation a

At room temperature, if $a \ll L$, edge effects are expected to be unimportant.

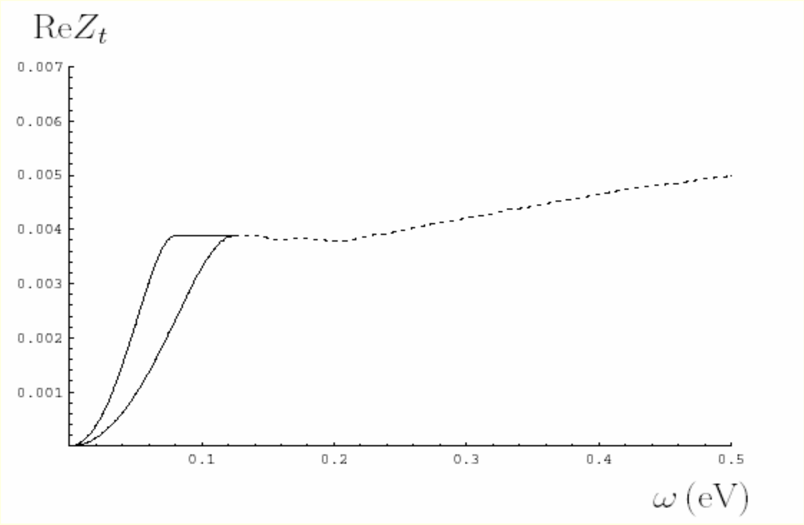
If $a \approx L$, as in MEMS, edge effects become important

At **low temperature**, the problem is more complicated. **Spatial correlation** of Johnson currents (anomalous skin effect) suppress fluctuations at small scales. If correlations extend over distances comparable to L, edge effects become important. (work in progress)

Modified expression of the infrared-optics impedance

V.B. Bezerra, G. Bimonte, G.L. Klimchitskaya, V.M. Mostepanenko and C. Romero, Eur.Phys. J. C52 (2007) 701.

$$Z_t' = \begin{cases} B \sin\left(\dfrac{\pi\omega^2}{2\beta^2}\right), & \omega \le \beta \\ B\ , \ \beta \le \omega \le 0.125\ eV \\ Y(\omega)\ , \ \omega \ge 0.125\ eV \end{cases}$$



Y(ω) stands for tabulated data, available for ω >0.125 eV We allowed 0.08 eV < β < 0.125 eV