THE TRONDHEIM GROUP (2008)

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Coworkers:
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Recent collaboration:
Stefan Yoshi Buhmann (Imperial College, London)
Stefan Scheel (Imperial College, London).

1 The Nernst theorem: Conductors

Lifshitz formula, nonmagnetic medium:

$$\mathbf{F}=\frac{\mathbf{T}}{\mathbf{2}\pi}{\sum_{\mathbf{m}=\mathbf{0}}^{\infty}}'\int_{\zeta_{\mathbf{m}}}^{\infty}\left[\ln(\mathbf{1}-\mathbf{A}\mathbf{e}^{-\mathbf{2}\kappa\mathbf{a}})+\ln(\mathbf{1}-\mathbf{B}\mathbf{e}^{-\mathbf{2}\kappa\mathbf{a}})\right]\kappa\mathbf{d}\kappa$$

$$\mathbf{A}=\left(\frac{\mathbf{s}-\varepsilon\mathbf{p}}{\mathbf{s}+\varepsilon\mathbf{p}}\right)^{\mathbf{2}}\quad(\mathbf{TM})$$

$$\mathbf{B}=\left(\frac{\mathbf{s}-\mathbf{p}}{\mathbf{s}+\mathbf{p}}\right)^{\mathbf{2}}\quad(\mathbf{TE})$$

$$\zeta_{\mathbf{m}}=\mathbf{2}\pi\mathbf{m}\mathbf{T},$$

$$\mathbf{s}=\sqrt{\varepsilon-\mathbf{1}+\mathbf{p}^{\mathbf{2}}},\quad\mathbf{p}=\frac{\kappa}{\zeta_{\mathbf{m}}}$$

Drude dispersion relation:

$$\varepsilon(\mathbf{i}\zeta) = \mathbf{1} + \frac{\omega_{\mathbf{p}}^{\mathbf{2}}}{\zeta(\zeta + \nu)}$$

Gold, at room temperature:

$$\omega_{\mathbf{p}} = \mathbf{9.03}\mathrm{eV}, \ \nu = \mathbf{34.5}\mathrm{meV}, \ \lambda_{\mathbf{p}} = \frac{\mathbf{2}\pi\mathbf{c}}{\omega_{\mathbf{p}}} = \mathbf{137.4}\mathrm{nm}$$

Drude good for $\zeta < 2 \times 10^{15}$ rad/s (Lambrecht-Reynaud 2000).

In principle (perfect lattice): Bloch-Grüneisen

$$\nu(\mathbf{T}) = \mathbf{0.0847}\left(\frac{\mathbf{T}}{\mathbf{\Theta}}\right)^{\mathbf{5}} \int_{\mathbf{0}}^{\mathbf{\Theta}/\mathbf{T}} \frac{\mathbf{x^5 e^x dx}}{(\mathbf{e^x} - \mathbf{1})^{\mathbf{2}}} \quad [\mathrm{eV}]$$

For gold, $\Theta = 175$ K.

$$\nu(\mathbf{T}) \propto \mathbf{T^5}, \ \mathbf{T} \to \mathbf{0}$$
$$\nu(\mathbf{T}) \propto \mathbf{T}, \ \mathbf{T} \text{ high}$$

Always impurities. Important point:

$$\zeta^{\mathbf{2}}[\varepsilon(\mathbf{i}\zeta) - \mathbf{1}] \to \mathbf{0}, \ \zeta \to \mathbf{0}$$

It implies that zero-frequency TE mode does not contribute to the Casimir force.

Others with same conclusion:

- **Jancovici-Samaj (2005)**
- **Buenzli-Martin (2005)**
- **Boström-Sernelius (2000)**

Linear dependence in T for the Casimir force as $T \to \infty$ is reduced by a factor of 2 from the behavior of an ideal metal (IM).

Plasma dispersion relation:

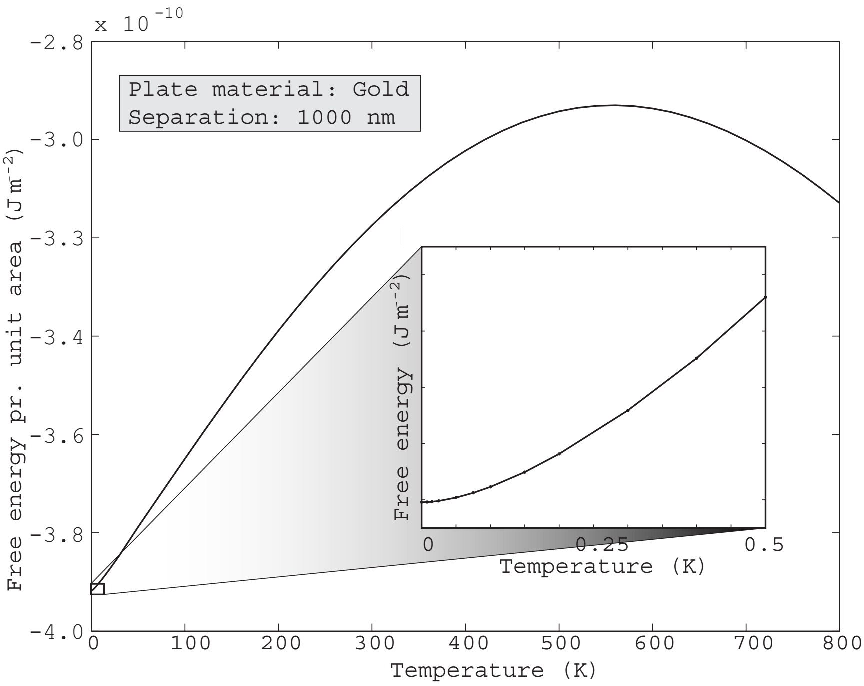
$$\varepsilon(\mathbf{i}\zeta) = \mathbf{1} + \frac{\omega_{\mathbf{p}}^{\mathbf{2}}}{\zeta^{\mathbf{2}}}$$

Leads to small temperature dependence in the Casimir force (correction $\propto T^4$).

If T increases from 300 K to 350 K:

- **a $= 0.2$ μm: Casimir pressure diminishes by 0.4%.**
- **a $= 2.0$ μm: Casimir pressure diminishes by 3.7%.**

$$\Delta F^{TE} = C_1 T^2 [1 - C_2 T^{1/2} + ...]$$



Recent publications (Nernst theorem)

- **I. Brevik, J. B. Aarseth,** *J. Phys. A* **390, 6589 (2006)**
- **J. S. Høye, I. Brevik, S. A. Ellingsen, J. B. Aarseth** *Phys. Rev. E* **75, 051127 (2007)**
- **I. Brevik, S. A. Ellingsen, J. S. Høye, K. A. Milton** *J. Phys. A* **41, 164017 (2008)**
- **S. A. Ellingsen,** *Phys. Rev. E* **78, 021120 (2008)**

Additional effects:

1. **Spatial dispersion (Sernelius 2005, Svetovoy & Esquivel 2005): Negligible contribution from**

zero-frequency mode. Nernst's theorem is satisfied.

2. **Anomalous skin effect (Esquivel-Svetovoy 2004, 2005). No contribution to Casimir force from zero TE mode.**

2 The Nernst theorem: Semiconductors

$$\varepsilon(i\zeta) = 1 + \frac{\bar{\varepsilon} - 1}{1 + \zeta^2/\omega_0^2} + \frac{4\pi\sigma}{\zeta}$$

Assume

$$0 < 4\pi\sigma \ll 2\pi T \ll \omega_0$$

If $\sigma \to 0$ as $T \to 0$ linearly or faster:

$$r_{TM}(i\zeta = 0) = 1, \quad \lim_{\zeta\to 0} r_{TM}(i\zeta) = \frac{\bar{\varepsilon} - 1}{\bar{\varepsilon} + 1},$$

then

$$F^{TM} = \frac{T}{16\pi a^2} \left[\mathbf{Li}_3(A_0) - \zeta(3)\right],$$

with

$$\mathbf{Li}_n(\xi) = \sum_{k=1}^{\infty} \frac{\xi^k}{k^n}, \quad A_0 = \left(\frac{\bar{\varepsilon} - 1}{\bar{\varepsilon} + 1}\right)^2.$$

Euler-Maclaurin formula:

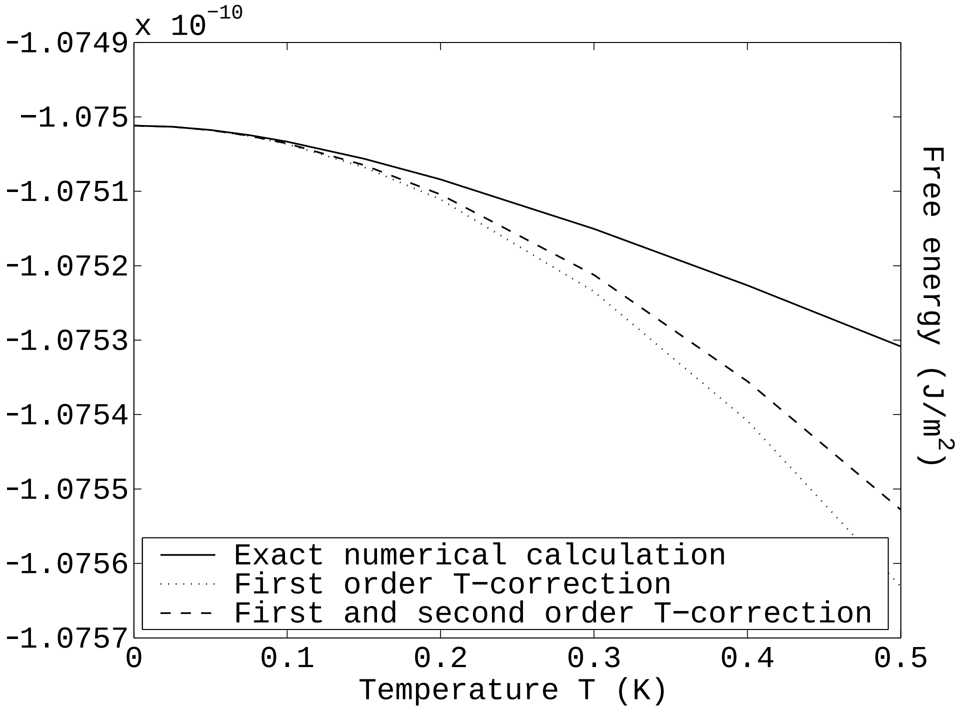
$$\Delta F_q = f(a,T) \left[\sum_{m=0}^{\infty}{}' - \int_0^{\infty} dm\right] g(m)$$

If σ is constant near $\zeta = 0$ and $T = 0$ (SI units):

$$\Delta F^{TM} = \frac{\pi^2 (k_B T)^2}{72\hbar(\sigma^{SI}/\epsilon_0) a^2} \left[1 - \frac{72\zeta(3) k_B T}{\pi^3 \hbar \sigma^{SI}/\epsilon_0}\right]$$

$$\Delta F^{TE} = \frac{\sigma^{SI} (k_B T)^2}{48\epsilon_0 \hbar c^2}(2\ln 2 - 1) - \frac{\zeta(3)(k_B T)^3}{8\pi\hbar^2 c^2}$$

Assumed σ small, T small, $\sigma a \ll 1$.



Reference:

S. A. Ellingsen, I. Brevik, J. S. Høye, K. A. Milton,
Phys. Rev. E **78, 021117 (2008).**

3 Real-frequency approach

Lifshitz:

$$F = \int_0^\infty d\omega \coth\left(\frac{\omega}{2T}\right) \Im\{\phi(\omega, T)\},$$

$$\phi(\omega, T) = \frac{1}{4\pi^2}\int_0^\infty dk_\perp k_\perp \sum_q \ln[1 - r_q^2 \exp(-2\kappa_0 a)].$$

Permittivities as above: metal/semiconductor.
Lifshitz variable

$$p = i\kappa_0/\omega, \quad \kappa_0 = (k_\perp^2 - \omega^2)^{1/2}.$$

Propagating waves: $p \in [1, 0]$
Evanescent waves: $p \in [0, i\infty]$. **Nernst holds for causal** $\varepsilon(\omega)$ **independent of** T **near** $T = 0$.

Results correspond to independent work by Intravaia & Henkel
(*J. Phys. A* **41 164018 (2008))**

Reference:
S. A. Ellingsen, *Phys. Rev. E* **78, 021120 (2008)**

4 Dispersive medium. Anomalies

Dispersive energy density

$$w_{disp} = \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^2k_\perp}{(2\pi)^2}\left[\frac{d(\varepsilon\omega)}{d\omega}\langle E^2\rangle_{\omega k} + \frac{d(\mu\omega)}{d\omega}\langle H^2\rangle_{\omega k}\right].$$

Dispersive energy refers thermodynamically to a non-closed system, different from the internal energy

$$U = \frac{\partial(\beta F)}{\partial\beta} = \frac{4\pi n^2}{\beta^3}{\sum_{m=0}^{\infty}}' m^2 \ln[1 - \exp(-4\pi naTm)]$$

Anomaly, for $D > 4$: generalization encountered for dispersive media.

I. Brevik, K, A. Milton, *Phys. Rev. E* **78, 011124 (2008)**

5 Relationship to the Feigel effect

Extraction of momentum from vacuum?

A. Feigel, *Phys. Rev. Lett.* **92, 020404 (2004)**

Magnetoelectric medium:

$$D = \varepsilon E + \chi B,$$
$$H = \chi^T E + \mu^{-1} B$$

O. J. Birkeland, I. Brevik, Phys. Rev. E 76, 066605 (2007)

6 Thermal Casimir-Polder force on molecules

Collaboration with Buhmann & Scheel (Imperial College London)

Full thermal theory:
Buhmann & Scheel *Phys. Rev. Lett.* **100, 253201 (2008)**

CP force differs from perturbative expansion of Lifshitz formula:
resonant forces due to excitation and de-exitation must be included.

Lifshitz formula + ground state polarisability *grossly overestimates* Casimir-Polder force on molecules with excited states close to ground state energy.

Work in progress (Details: Stefan Buhmann's talk).