

Quantum Fluctuations on Low Dimensional Topological Defects

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- 1 One-loop soliton mass shifts
- 2 Brief history
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One-loop soliton mass shifts

Object to be computed:

$$\text{Tr}K^{\frac{1}{2}} = \text{trTr}_{L^2}K^{\frac{1}{2}}$$

$$K = -\Delta + c^2 + Q(x) \cdot \nabla + V(x) = K_0 + Q(x) \cdot \nabla + V(x)$$

acting on (PBC boundary conditions)

$$\bigoplus_{a=1}^N L_a^2 \left(\bigotimes_{i=1}^d S_i^1 \right)$$

Heat kernel/Zeta function regularization methods

Zeta function regularization

$$\zeta_K(s) = \text{Tr} K^{-s}$$

Heat kernel, heat function, Mellin transform

$$\left(\frac{\partial}{\partial \beta} + K \right) G_K(x, y; \beta) = 0 \quad , \quad G_K(x, y; 0) = \delta(x - y)$$

$$\text{Tr} e^{-\beta K} = \int dx G_K(x, x; \beta)$$

$$\zeta_K(s) = \frac{1}{\Gamma(s)} \cdot \int_0^\infty d\beta \beta^{s-1} \text{Tr} e^{-\beta K}$$

Heat kernel/Zeta function regularization methods

High-temperature asymptotic expansion

$$G_K(x, y; \beta) = G_{K_0}(x, y; \beta) \sum_{n=0}^{\infty} c_n(x, y) \beta^n$$

$$G_{K_0}(x, y; 0) = \delta(x - y)$$

$$\sum_{c=1}^N [n \cdot \delta^{ac} + (x_k - y_k) \cdot (\delta^{ac} \partial_k - \frac{1}{2} Q_k^{ac}(\vec{x}))] c_n^{cb}(\vec{x}, \vec{y}; K)$$

$$= \sum_{c=1}^N [\delta^{ac} \cdot \Delta - Q_k^{ac}(\vec{x}) \partial_k - V^{ac}(\vec{x})] c_{n-1}^{cb}(\vec{x}, \vec{y}; K)$$

$$c_0^{ab}(\vec{x}, \vec{y}; K) = \delta^{ab}$$

Heat kernel/Zeta function regularization methods

High-temperature asymptotic expansion

$$\mathrm{Tr} e^{-\beta K} = \mathrm{Tr} e^{-\beta K_0} \sum_{n=1}^{\infty} \beta^n c_n \quad , \quad c_n^{ab} = \int dx c_n^{ab}(x, x)$$

Renormalizations

Zero point energy renormalization. Subtract

$$\text{Tr}K_0^{-s} = \zeta_{K_0}(s)$$

Mass renormalization. Subtract

$$\text{Tr}K_0^{-s-1} = \zeta_{K_0}(s+1)$$

- Take the limit $s \rightarrow -\frac{1}{2}$

Conceptual kinship to the Casimir effect

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Bosonic kinks

$$K = -\frac{d^2}{dx^2} + V(x) \quad , \quad L^2(S^1)$$

A. A. Izquierdo, M. A. G. Leon, W. G. Fuertes, and J. M. Guilarte

Generalized zeta functions and one-loop corrections to quantum kink masses

Nuclear Physics **B635**[PM](2002) 525-557

SUSY kinks

M. Bordag, A. Goldhaber, P. van Nieuwenhuizen, and D. Vassilevich

Heat kernels and zeta function regularization for the mass of the SUSY kink

Physical Review **D66** (2002) 125014

Multicomponent bosonic kinks

$$K_{ab} = -\frac{d^2}{dx^2} \delta_{ab} + V_{ab}(x) \quad , \quad \bigoplus_{a=1}^N L^2(S^1)$$

A. A. Izquierdo, M. A. G. Leon, W. G. Fuertes, and J. M. Guilarte

Semi-classical mass of quantum k-component topological kinks
Nuclear Physics **B638**[PM](2002) 378-404

A. A. Izquierdo, M. A. G. Leon, W. G. Fuertes, and J. M. Guilarte

One-loop corrections to classical masses of kink families
Nuclear Physics **B681**[PM](2004) 163-194

Fermionic fluctuations around a Nielsen-Olesen vortex

M. Bordag, and I. Drozdov

Fermionic vacuum energy from a Nielsen-Olesen vortex

Physical Review **D68**(2003) 065026

$\mathcal{N} = 2$ SUSY vortex

D. Vassilevich

Quantum corrections to the mass of the supersymmetric vortex

Physical Review **D68** (2003) 045005

A. Rebhan, P. van Nieuwenhuizen, and R. Wimmer

Nonvanishing quantum correction to the mass and central charge of the $\mathcal{N} = 2$ vortex and BPS saturation

Nuclear Physics **B679** (2004) 382-394

Bosonic BPS magnetic flux lines

$$\bigoplus_{a=1}^4 L_a^2(S^1 \otimes S^1)$$

A. A. Izquierdo, M. T. Mayado, W. G. Fuertes, and J. M. Guilarte
Quantum corrections to the mass of self-dual vortices

Physical Review **D70** (2004) 061702(R)

A. A. Izquierdo, M. T. Mayado, W. G. Fuertes, and J. M. Guilarte
Quantum oscillations of self-dual Abrikosov-Nielsen-Olesen vortices

Physical Review **D71** (2005) 125010

A. A. Izquierdo, M. T. Mayado, W. G. Fuertes, and J. M. Guilarte
One-loop mass shift formula for kinks and self-dual vortices

Journal of Physics **A39** (2006) 6463-6472

Semi-local strings

$$\bigoplus_{a=1}^6 L_a^2(S^1 \otimes S^1)$$

A. A. Izquierdo, M. T. Mayado, W. G. Fuertes, and J. M. Guilarte
One-loop corrections to the mass of self-dual semi-local topological solitons

Nuclear Physics **B797**[PM] (2008) 431-463

A. A. Izquierdo, M. T. Mayado, W. G. Fuertes, and J. M. Guilarte
One-loop fluctuations of semi-local self-dual vortices

Journal of Physics **A41** (2008) 164050

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Analysis of the impact of changing the boundary conditions

Heat and zeta functions with Dirichlet and Robin boundary conditions.

Better calibration of the errors coming from rejecting terms in the asymptotic expansion

Improvement of symbolic and numerical programs implemented in Mathematica to obtain the Seeley coefficients

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Future objectives

Application of the method to study kink fluctuations in non-linear sigma models

Study in this framework of non-commutative and SUSY non-commutative kinks

What about BPS magnetic monopoles?