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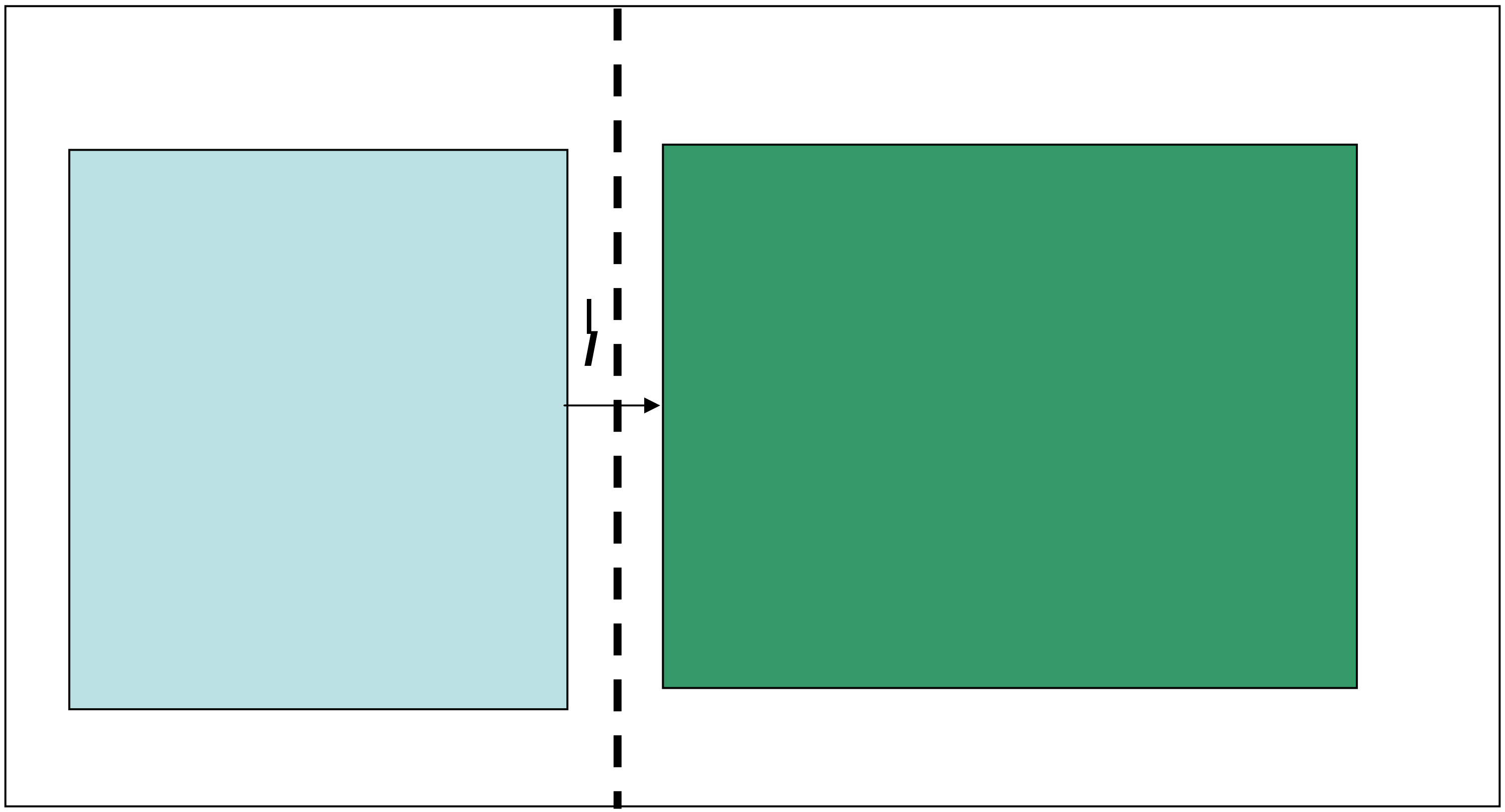
Part 1. Casimir-Lifshitz Forces and Entropy

Interaction between two bodies



$$f = \sigma_{xx}$$

What theory can say about Entropy?

The theory gives the ONLY one quantity

- the force: $f(T,l)$. Thermodynamics:

$$f(T,l) = -\left(\frac{\partial F(T,l)}{\partial l}\right)_T$$

$$\left(\frac{\partial S(T,l)}{\partial l}\right)_T = \left(\frac{\partial f(T,l)}{\partial T}\right)_l$$

$$\Delta S = S(T,l) - S(T,l_0) = \int_{l_0}^{l} \left(\frac{\partial f(T,l)}{\partial T}\right)_l dl$$

Nernst’s theorem

According to Nernst's theorem (it is valid for ordered bodies - quantum fluids, ideal crystals):

$$S(T,l)|_l \to 0, T \to 0$$

Consequences of Nernst's theorem

It MUST be

$$\left(\frac{\partial S(T,l)}{\partial l}\right)_T = \left(\frac{\partial f(T,l)}{\partial T}\right)_l \to 0$$

$$T \to 0,\, l = const$$

$$\Delta S \equiv S(T,l) - S(T,l_0) \propto \left(\frac{\partial f(T,l)}{\partial T}\right)_l \to 0$$

No information about signs at finite T !

Is it important to keep $l = const$? **Yes**!

Instructive example. Degenerate ideal gas.

Entropy per atom:

$$S(T,V) = S\left(TV^{2/3} / N^{2/3}\right)$$

Let: $T \to 0$ **but**

$$V \to \infty, TV^{2/3} \to const$$

Then $S(T,V) \neq 0$ at $T \to 0$

$l \to \infty$ means $V \to \infty$!!!

Popular definition of “Casimir entropy”

$$l_0 \to \infty$$

$$S_C(T,l) \equiv -\int_l^{\infty} \left(\frac{\partial f(T,l)}{\partial T} \right)_l dl$$

$$= S(T,l) - S(T,\infty)$$

$$S_C(T,l) > 0\ ?$$

$$T \to 0 : S_C(T,l)\,|_l \to -S(T,\infty) \to 0\ ?$$

Why? **No obligations.**

About the sign of "Casimir entropy"

$$S_C(T,l) \equiv S(T,l) - S(T,\infty)$$

A point : temperature dependences of

$S(T,l)$ and $S(T,\infty)$ at $T \to 0$

can be different.

Large l can give finite contribution.

If at $T \to 0 : \left|S_C(T,l)\right| >> \left|\Delta S(T,l,l_0)\right|,$

$$S_C(T,l) \approx -S(T,\infty) < 0$$

S_C is negative just due to Nernst's theorem!

When "Casimir entropy" can be negative ?

Let at large l and small T

the main contribution in f

is from $\omega \sim \omega_0(T, l)$. If at $l \to \infty$

$\hbar\omega_0 \leq k_B T$, $S(T, \infty)$ will be large

and even finite at $T = 0$.

Then $S_C < 0$.

Conclusion:

The “Casimir entropy” S_C is not the simplest characteristic of thermal properties of the Casimir-Lifshitz forces.

Examples

$$T \to 0$$

1. Ideal metall. Anamalous skin effect theory.

$$\frac{\partial S}{\partial l} \sim \Delta S \propto T, S_C \propto T^{2/3}$$

$$|S_C| >> |\Delta S|, S_C < 0$$

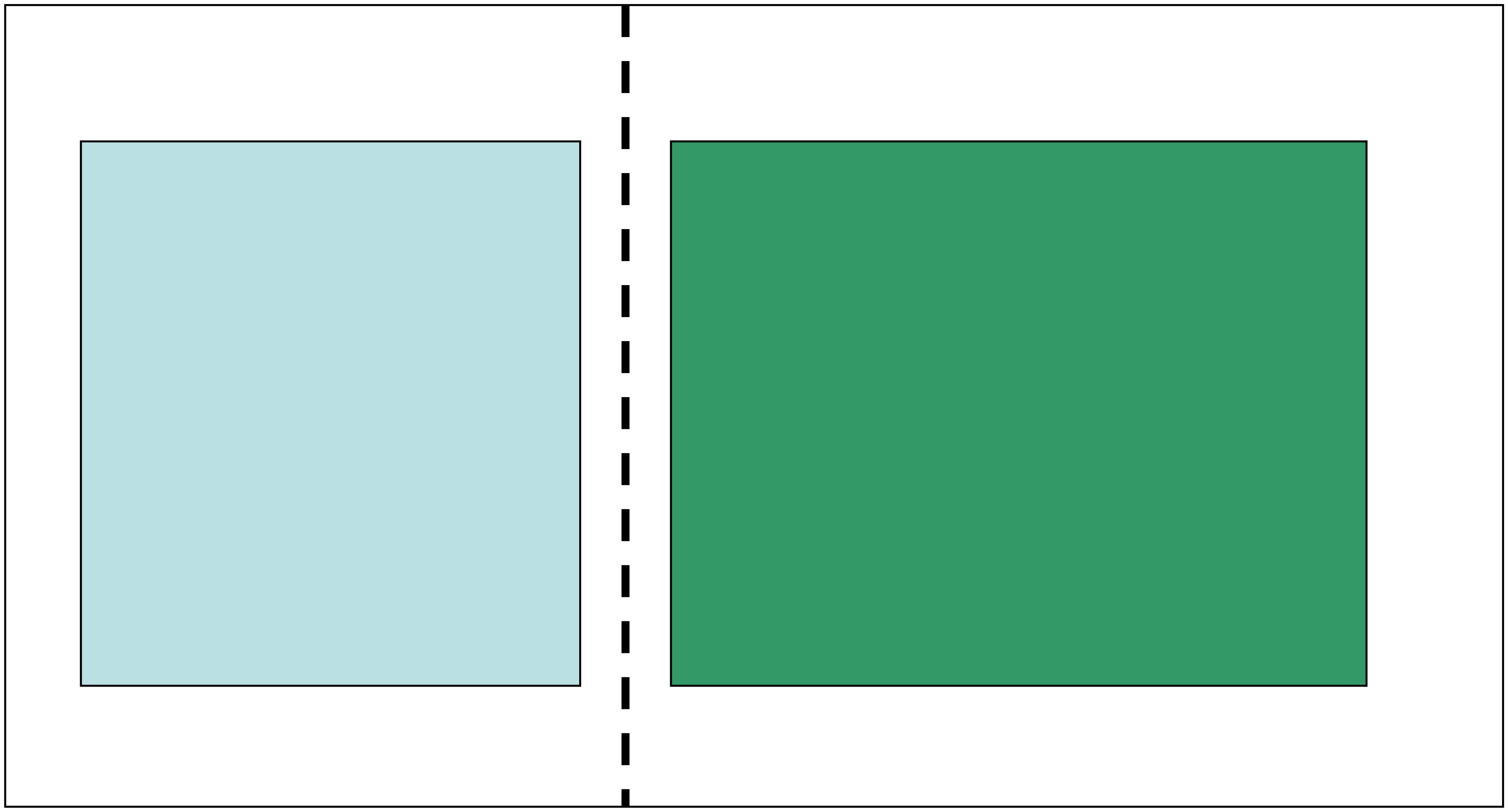
2. Carriers localized by disorder.

$$\frac{\partial S}{\partial l} \sim \Delta S \sim S_C \to const$$

Sign S_C cannot be predicted. (Actually $S_C > 0$.)

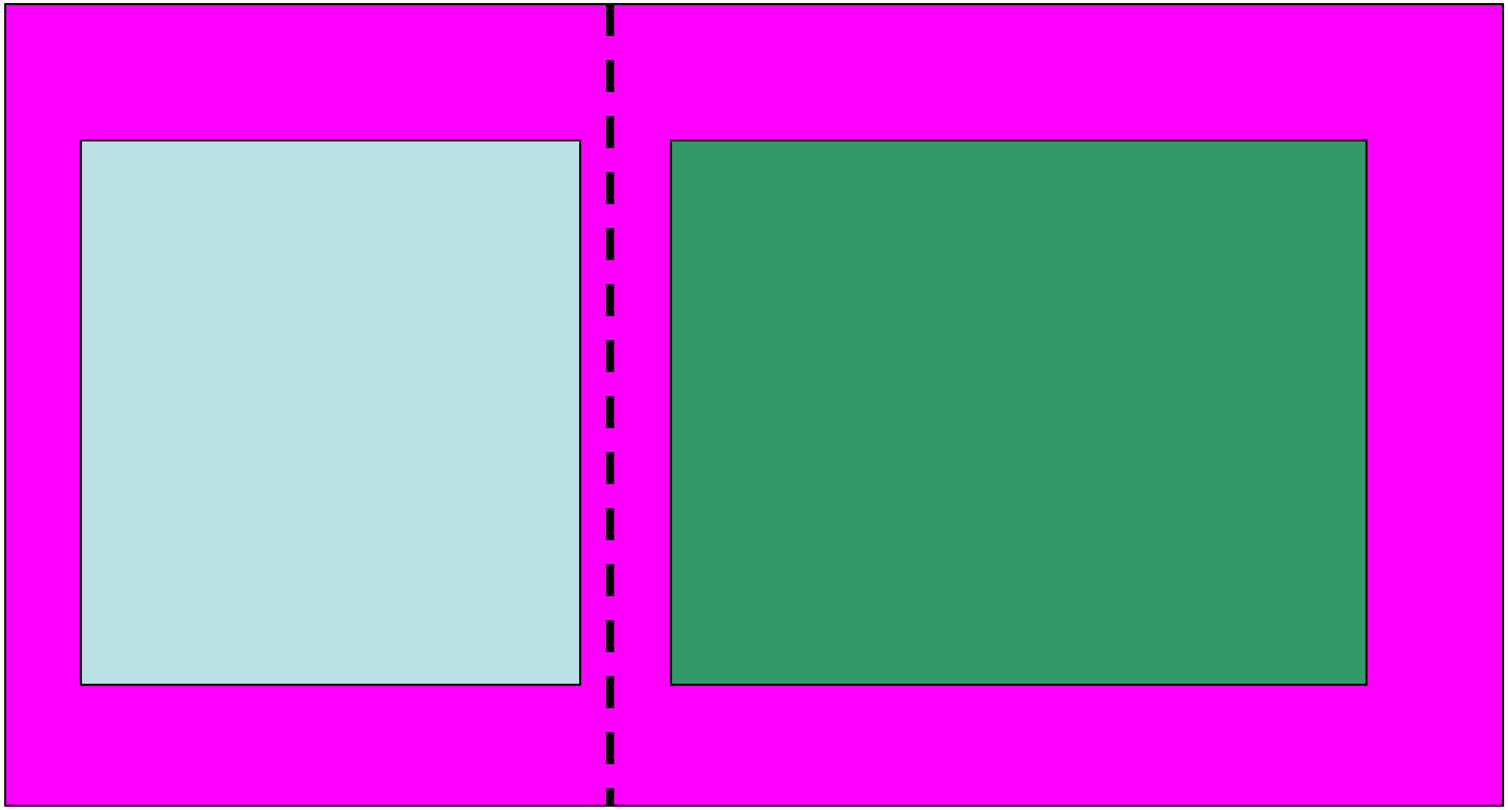
Part 2. Interaction through medium

Interaction through vacuum



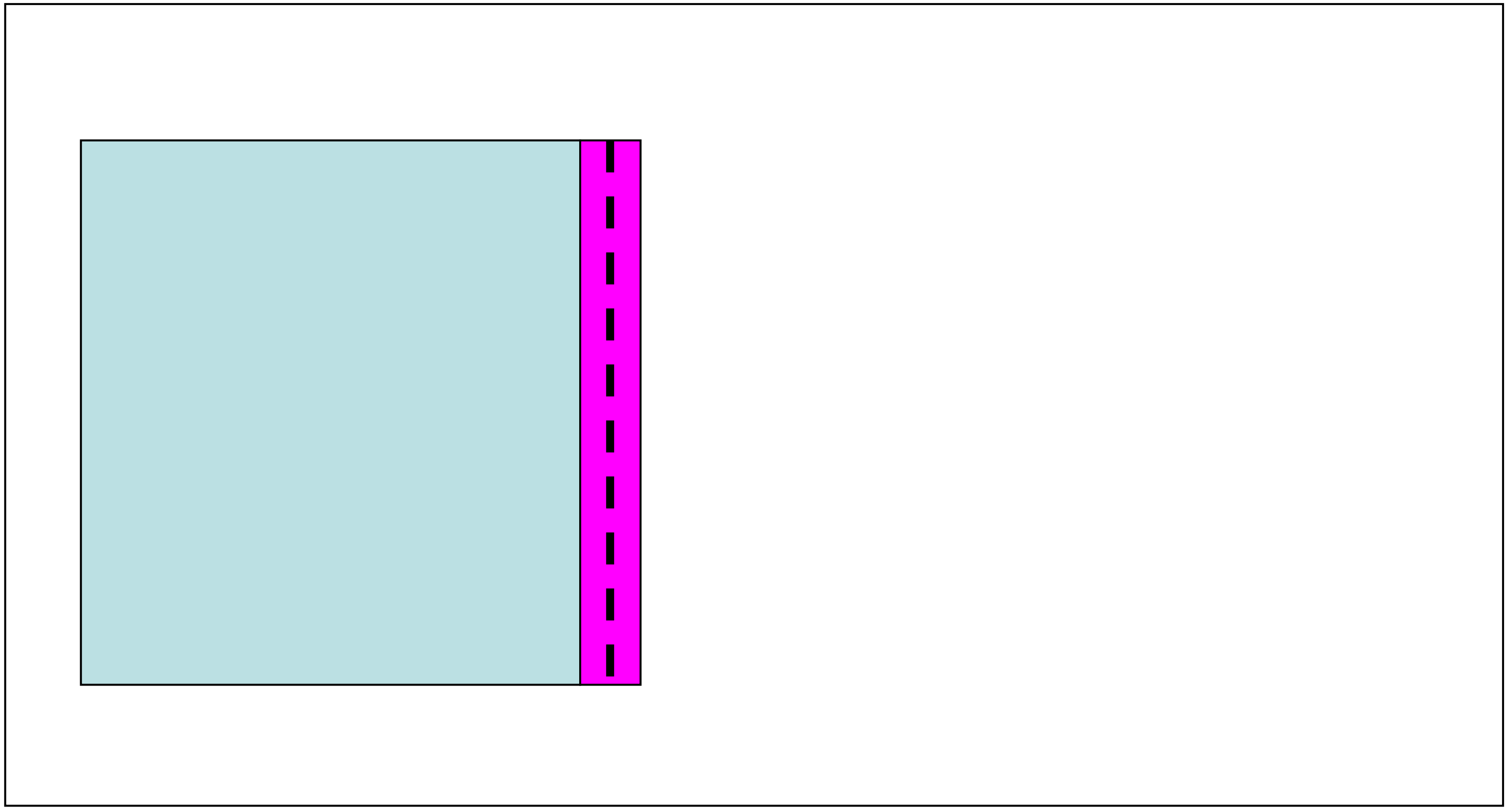
σ_{xx}

Interaction through medium



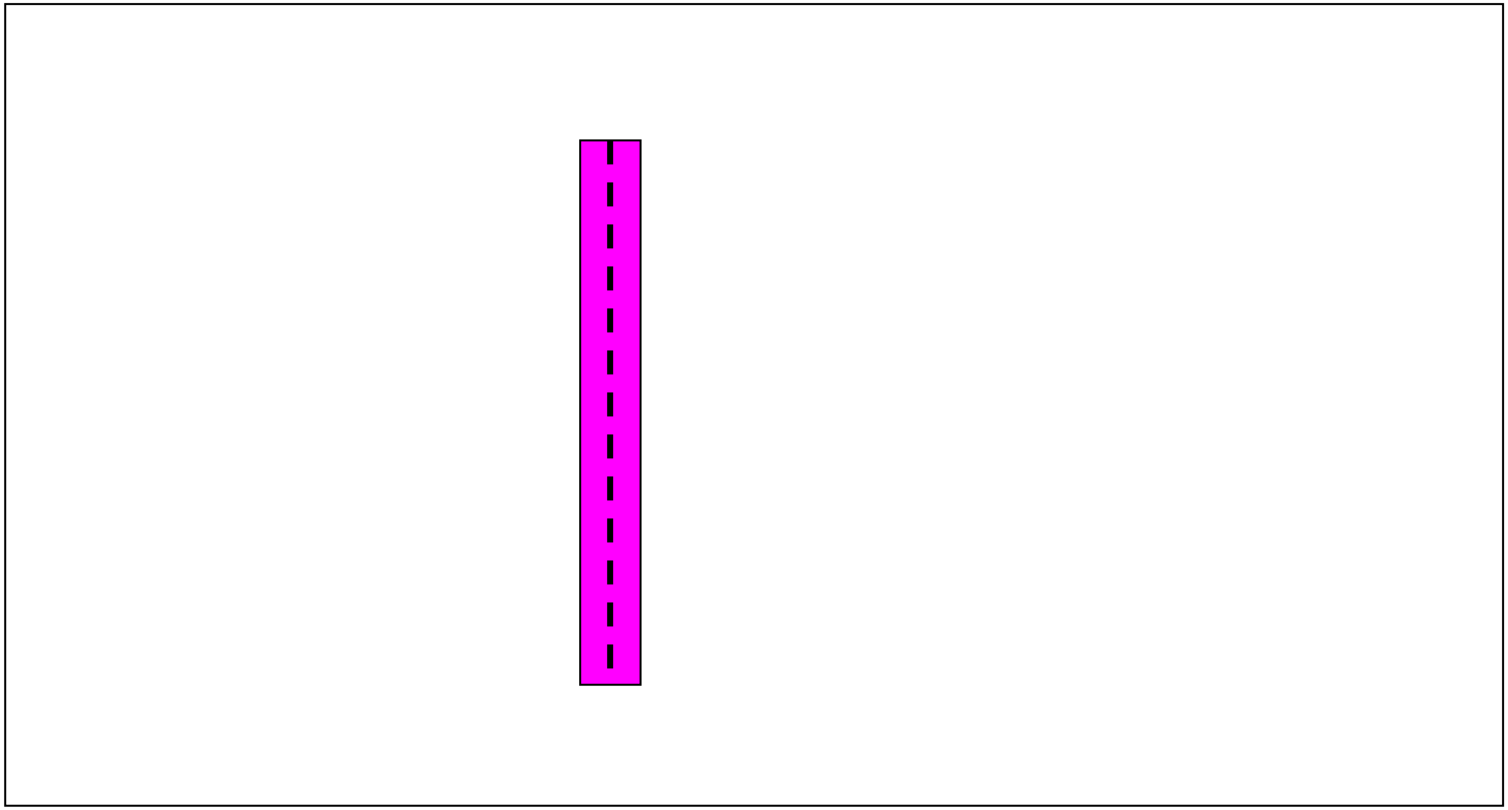
$$\sigma_{xx} = ?$$

Problem of a film



$$\mu(d) = \sigma_{xx}(d)$$

Problem of a film in vacuum



$$\mu(d) = \mu_\infty + \delta\mu(d)$$

Generalization to electrolytes?