The Casimir force in Twente

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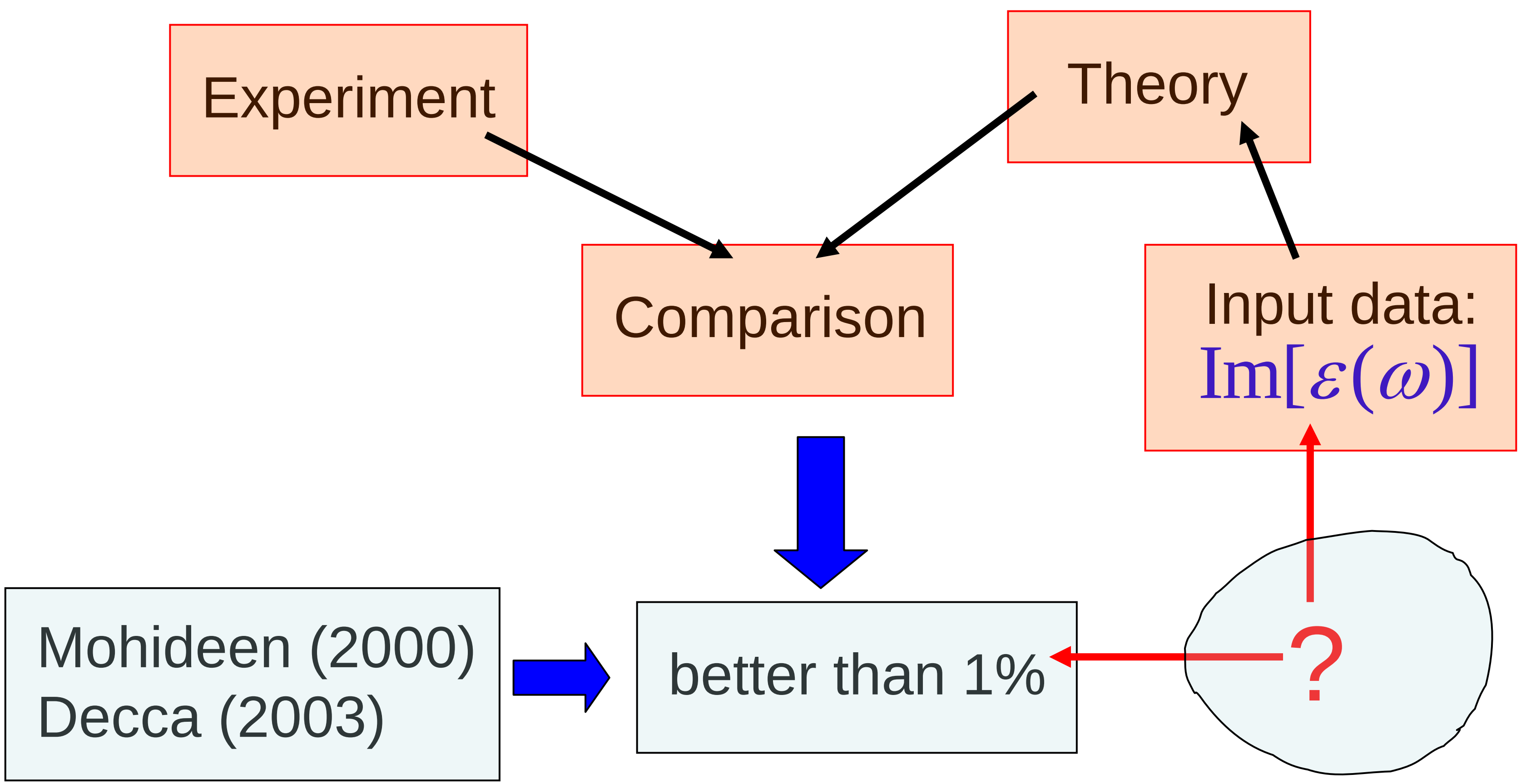
The fields of activity

- Optical properties of Au films
  (in collaboration with U. Groningen)
- Spatial dispersion and the Casimir force
  (in collaboration with U. Trento)
- The force in external fields
  (expected collaboration with U. Mexico)

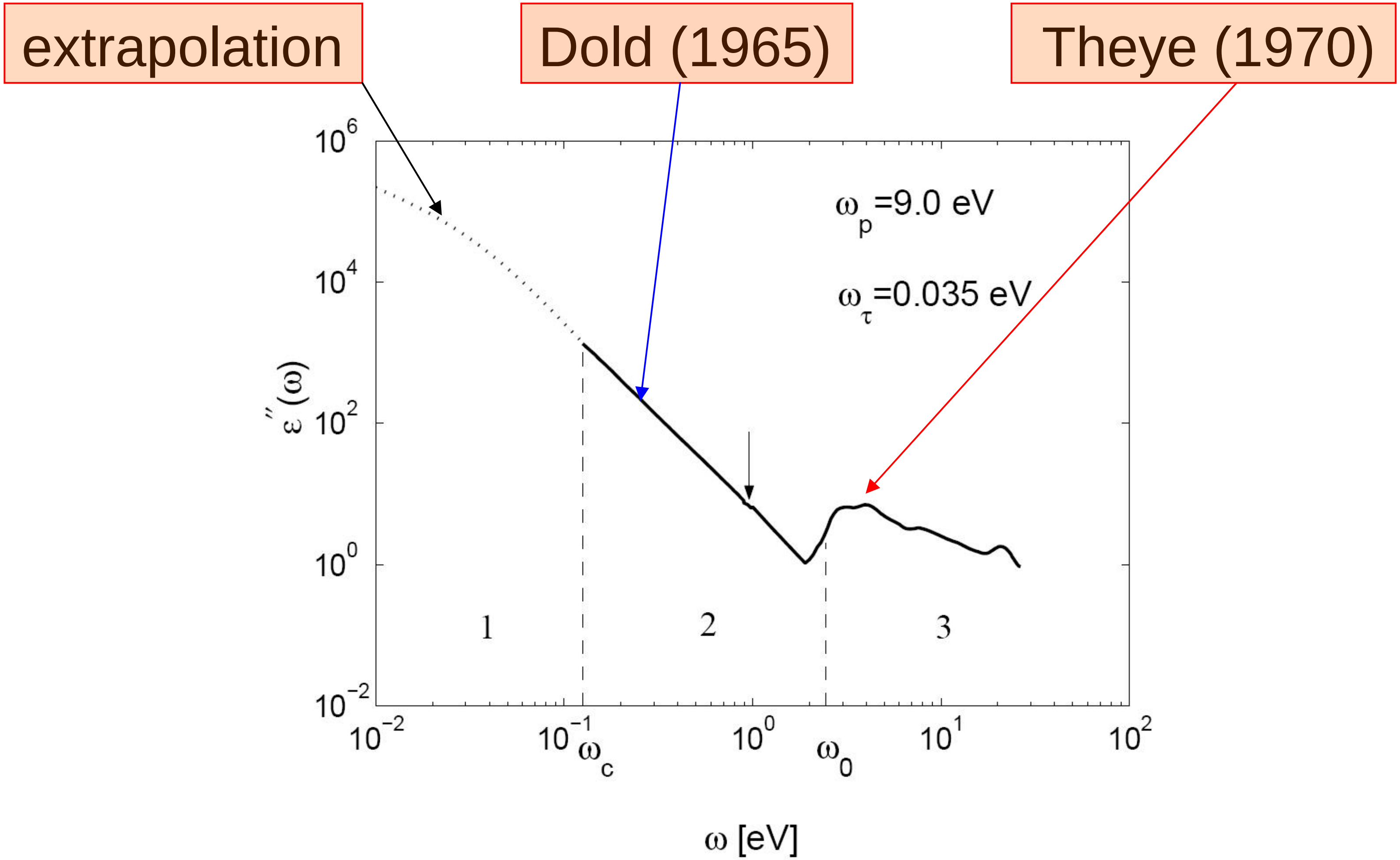
Optical properties of Au films

Collaborators: P. van Zwol, G. Palasantzas, J. De Hosson

How good we understand the Casimir force?

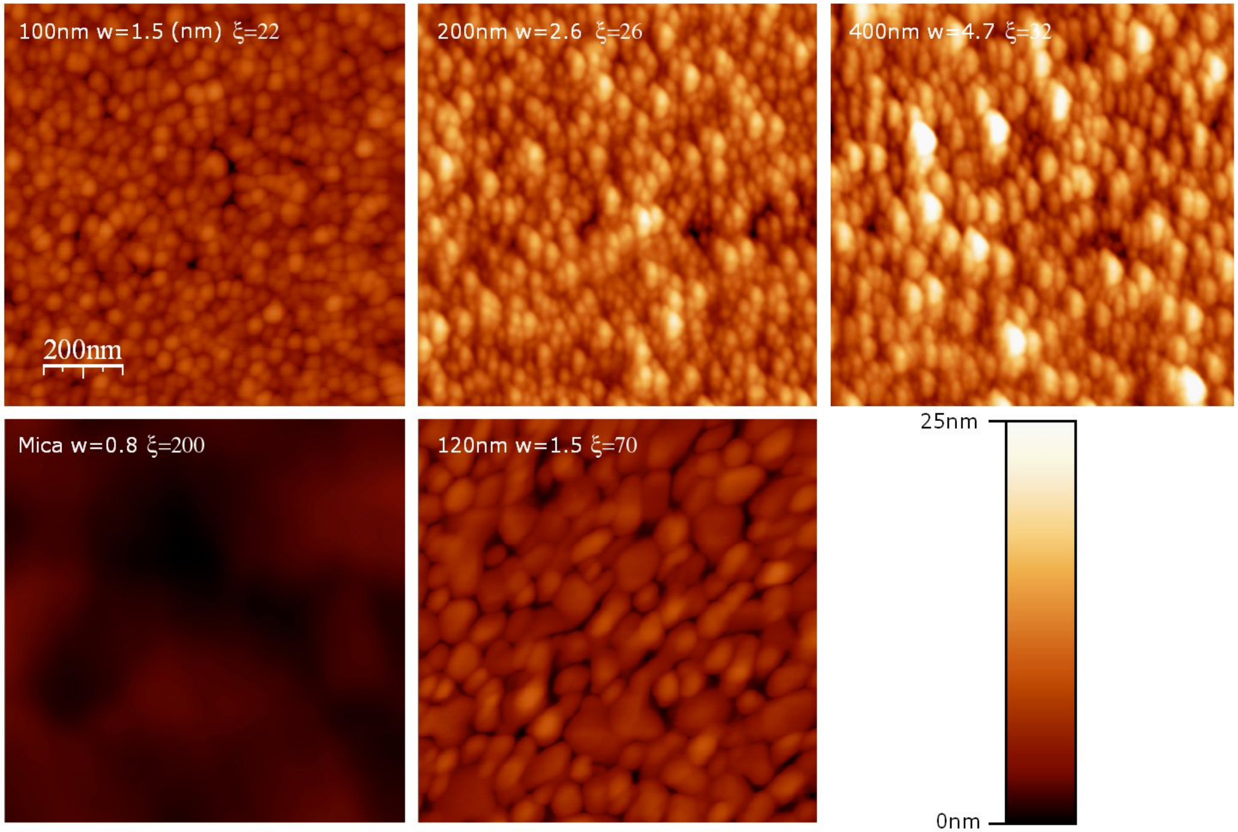


$\varepsilon''(\omega)$ used for comparison with the experiment



Imaginary material close to a perfect single crystal

Realistic materials can be very different

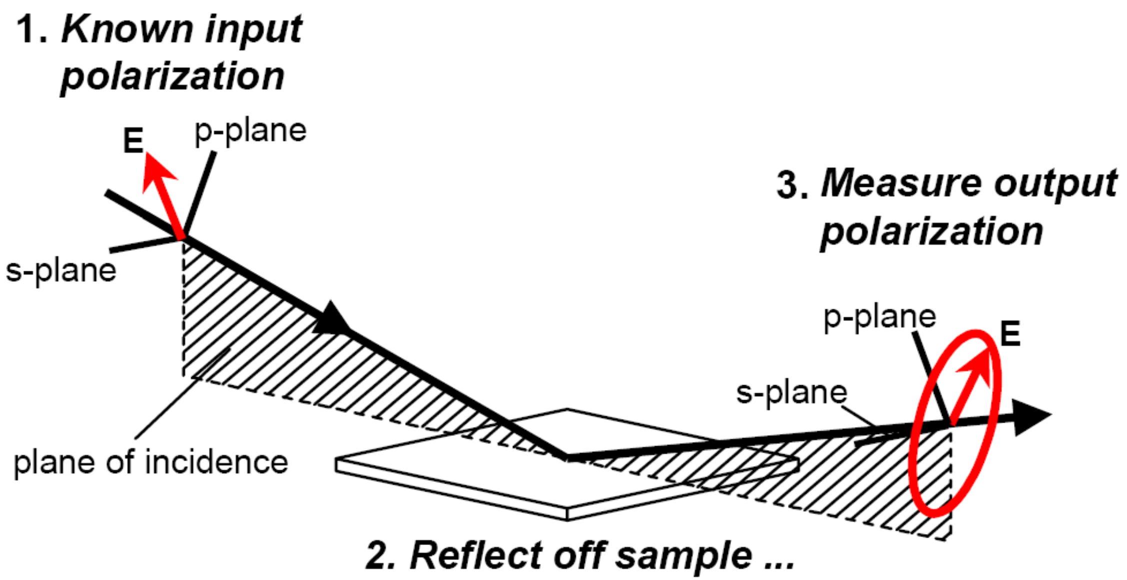


Au films deposited at different conditions

J. A. Woollam Co., Inc.

VUV-VASE® 0.14 – 1.7 μm

IR-VASE® 1.9 – 33 μm

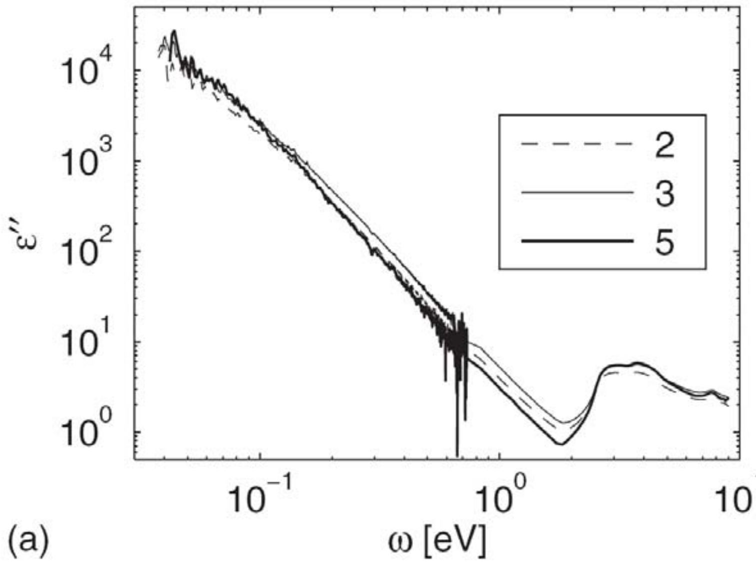


Measures $\rho = \frac{R_P}{R_S} = \tan(\psi)e^{i\Delta}$ **or** $\varepsilon = \sin^2\vartheta\left[1 + \tan^2\vartheta\left(\frac{1-\rho}{1+\rho}\right)^2\right]$

TABLE I. Dielectric function for different samples at fixed wavelengths $\lambda = 1, 5, 10\ \mu m$.

Sample	$\lambda = 1\ \mu m$	$\lambda = 5\ \mu m$	$\lambda = 10\ \mu m$
1, 400 nm/Si	$-29.7+i2.1$	$-805.9+i185.4$	$-2605.1+i1096.3$
2, 200 nm/Si	$-31.9+i2.3$	$-855.9+i195.8$	$-2778.6+i1212.0$
3, 100 nm/Si	$-39.1+i2.9$	$-1025.2+i264.8$	$-3349.0+i1574.8$
4, 120 nm/Si	$-43.8+i2.6$	$-1166.9+i213.9$	$-3957.2+i1500.1$
5, 120 nm/mica	$-40.7+i1.7$	$-1120.2+i178.1$	$-4085.4+i1440.3$

Considerable sample dependence



Drude parameters

1. joint fit $\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \omega_\tau^2}, \quad \varepsilon''(\omega) = \frac{\omega_p^2 \omega_\tau}{\omega(\omega^2 + \omega_\tau^2)}$

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2. joint fit $y = \omega / \omega_\tau$

$$n(\omega) = \frac{\omega_p}{\sqrt{2}\omega_\tau} \frac{1}{\sqrt{1+y^2}} \left[1 + \frac{\sqrt{1+y^2}}{y}\right]^{-1/2} \qquad k(\omega) = \frac{\omega_p}{\sqrt{2}\omega_\tau} \frac{1}{\sqrt{1+y^2}} \left[1 + \frac{\sqrt{1+y^2}}{y}\right]^{1/2}$$

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3. K-K relation $\varepsilon'(\omega) - 1 = \frac{2}{\pi} P \int_0^\infty dx \frac{x\varepsilon''(x)}{x^2 - \omega^2}$

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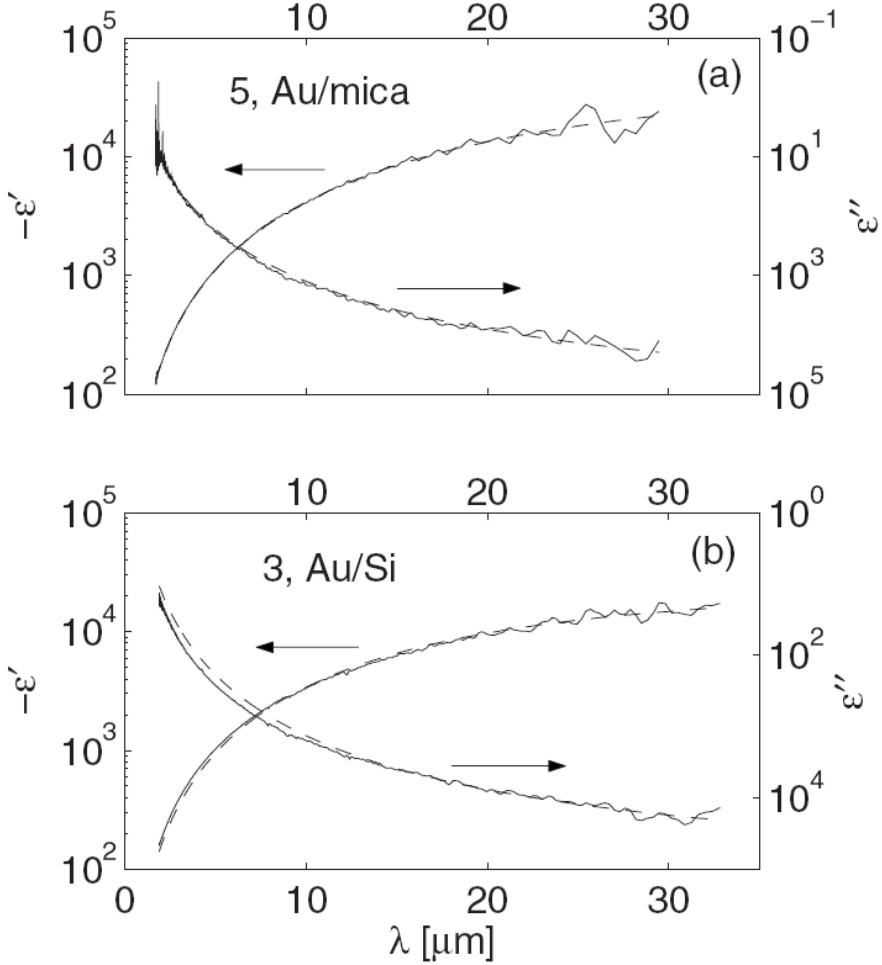
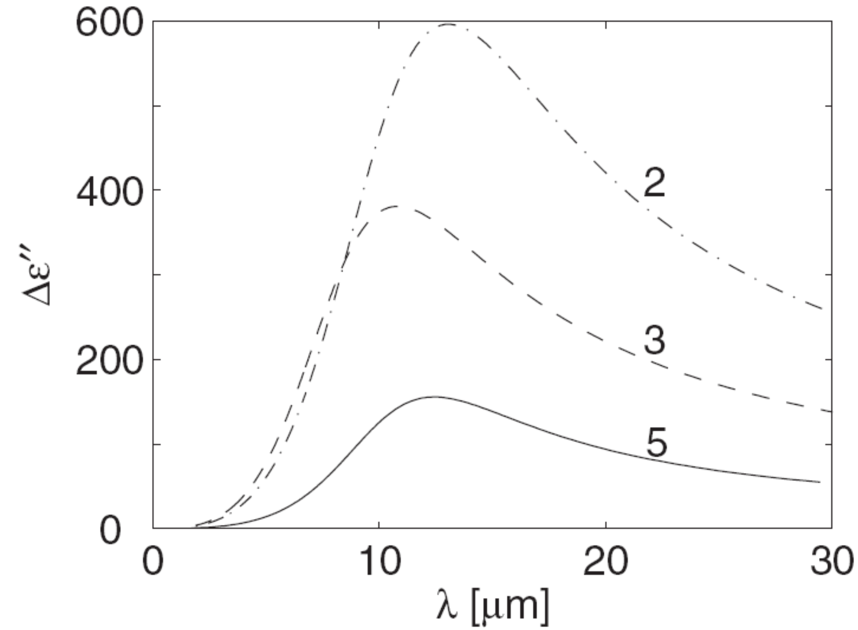
4. K-K relation $n(\omega) - 1 = \frac{2}{\pi} P \int_0^\infty dx \frac{xk(x)}{x^2 - \omega^2}$

Sample	Parameter	Joint ε', ε''	Joint n, k	KK ε'	KK n	Average
1	ω_p [eV]	6.70	6.87	6.88	6.83	6.82 ± 0.08
400 nm/Si	ω_τ [meV]	38.4	43.3	40.2	39.9	40.5 ± 2.1
2	ω_p	6.78	7.04	6.69	6.80	6.83 ± 0.15
200 nm/Si	ω_τ	40.7	45.3	36.1	36.0	39.5 ± 4.4
3	ω_p	7.79	7.94	7.80	7.84	7.84 ± 0.07
100 nm/Si	ω_τ	48.8	52.0	47.9	47.4	49.0 ± 2.1
4	ω_p	7.90	8.24	7.95	7.90	8.00 ± 0.16
120 nm/Si	ω_τ	37.1	41.4	35.2	29.2	35.7 ± 5.1
5	ω_p	8.37	8.41	8.27	8.46	8.38 ± 0.08
120 nm/mica	ω_τ	37.1	37.7	34.5	39.1	37.1 ± 1.9

Different methods give close parameters but variation exceeds the statistical errors

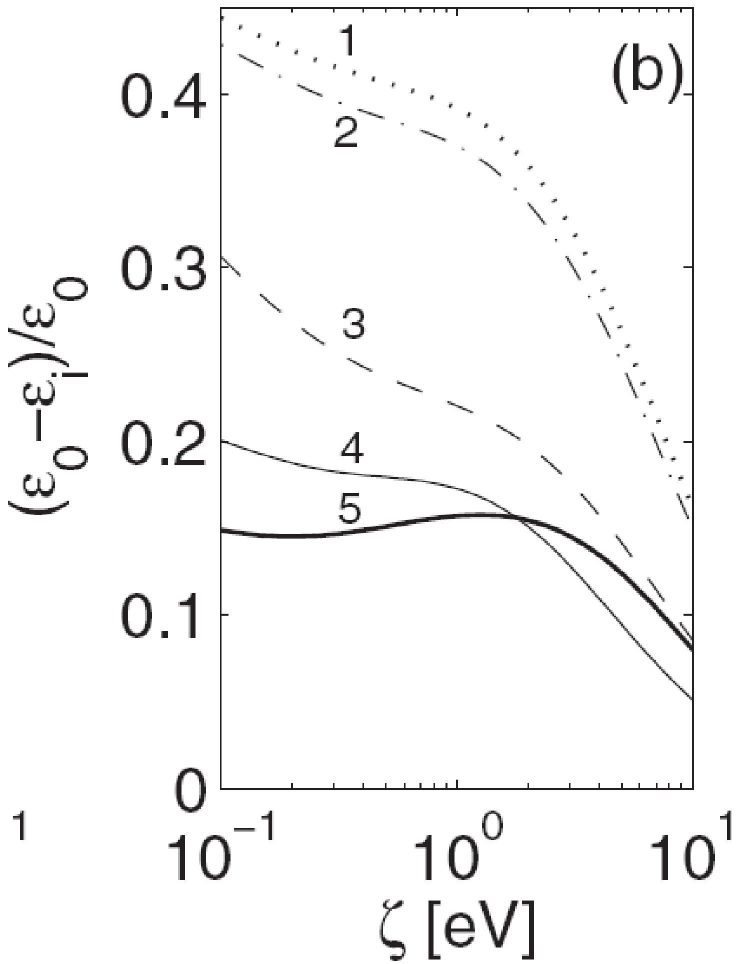




Deviations from Drude

Quality of the fits

ε at imaginary frequencies



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$$\varepsilon(i\zeta) = 1 + \frac{2}{\pi}\int_0^\infty d\omega \frac{\omega\varepsilon''(\omega)}{\zeta^2+\omega^2}$$

ε_0 handbook data

ε_i our samples

Characteristic frequency

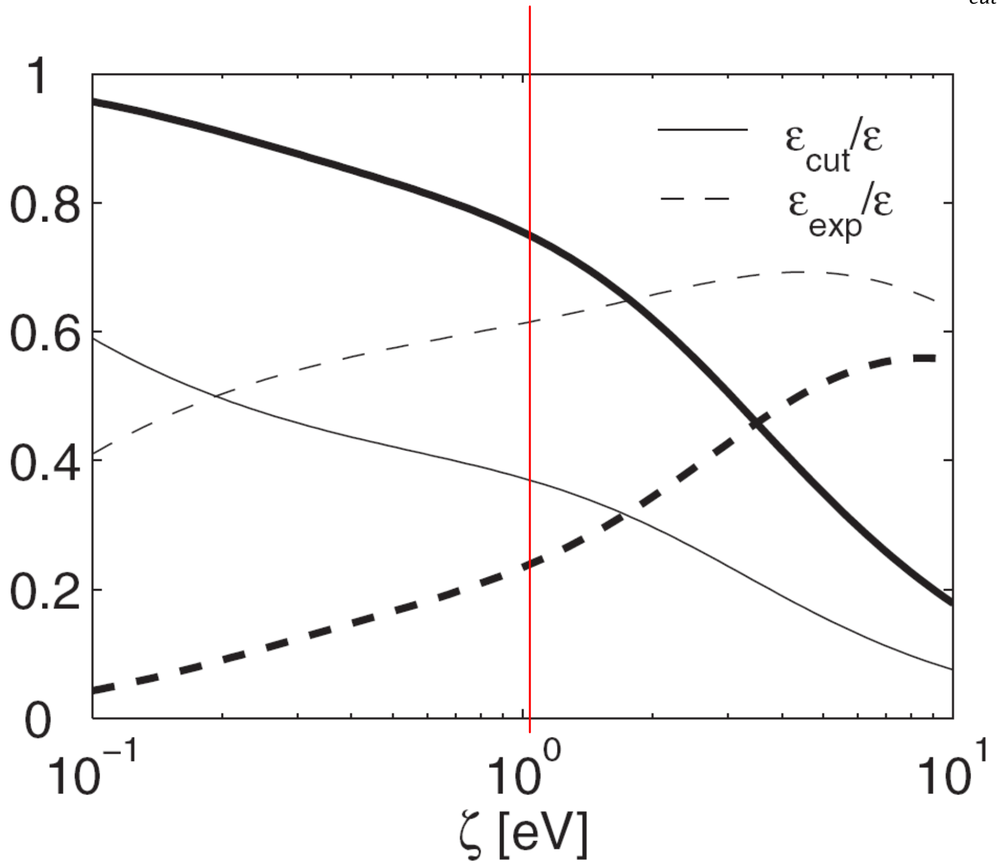
$$\zeta_c = \frac{c}{2a} = 0.988\left(\frac{100\ nm}{a}\right) \quad eV$$

Role of low-frequency cutoff

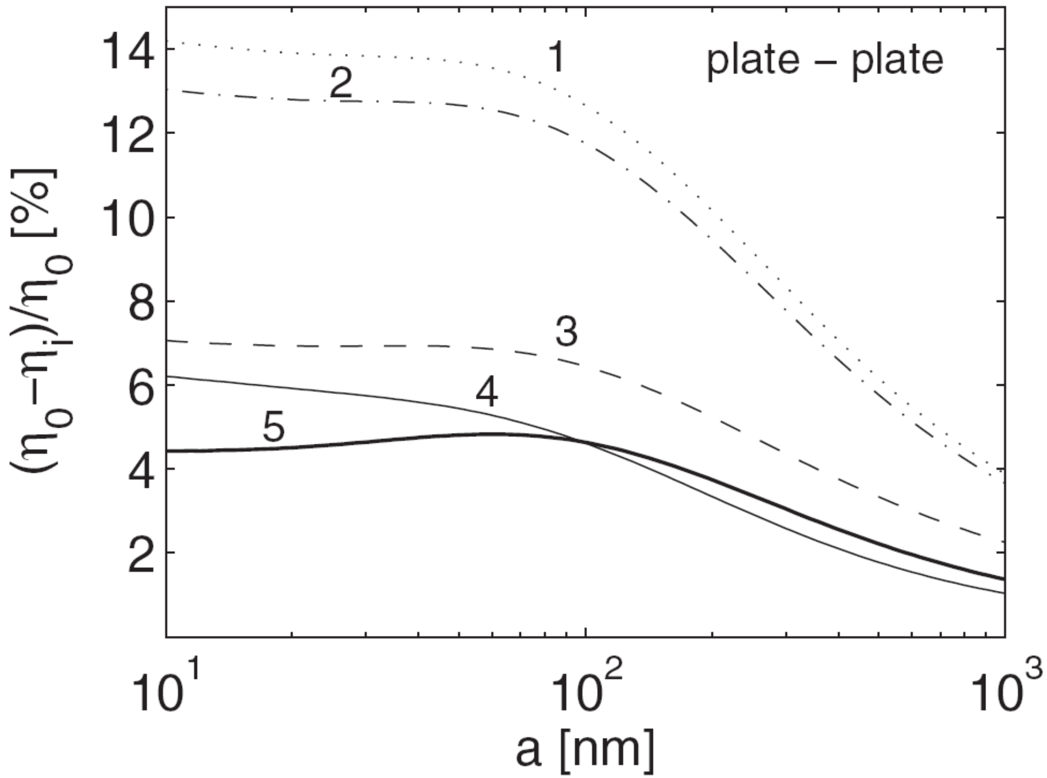
$$\lambda_{cut} = \begin{cases} 33\ \mu m & \text{our} \\ 10\ \mu m & \text{Palic} \end{cases}$$

$$\varepsilon = 1 + \varepsilon_{cut} + \varepsilon_{exp} = \frac{2}{\pi}\int_0^{\infty} d\omega \frac{\omega\varepsilon''(\omega)}{\zeta^2+\omega^2}$$

$$\varepsilon_{cut} \sim \int_0^{\omega_{cut}} , \quad \varepsilon_{exp} \sim \int_{\omega_{cut}}^{\infty}$$



Variation of the Casimir force



Reduction factor $$\eta = \frac{F(a)}{F_C(a)}, \qquad F_C(a) = \frac{\pi^2 \hbar c}{240 a^4}$$

Errors in the Drude parameters – 1% in force variation

- **Material properties have to be measured if you aim at precision better than 10%**
- **1% is the lower limit for the force prediction**
- **Our results are in agreement with the literature analysis by Pirozhenko, Lambrecht and Svetovoy**

No experiment to use as a reference

No two experiments reliably comparable to each other

Role of Spatial Dispersion $\varepsilon(\omega, k)$

Two situation when SP cannot be ignored:

1. metals with low defect density at low T;

2. poor conductors.

Why local approximation is usually works well?

For metals at $T > T_D$: short Debye length, l_D ~1 Å

For dielectrics: no free charges, $l_D \rightarrow \infty$

Metals at T << T_D

small

Relaxation frequency $$\omega_\tau(T) = \omega_\tau(0) + C_e T^2 + C_{ph} T^5$$

Mean free path $$l(T) = \frac{v_F}{\omega_\tau(T)}$$

Inevitably $$l(T) >> \delta = \frac{c}{\omega_p}$$ at some small T

Anomalous skin effect

Poor conductors

Test charge in plasma:

$$\operatorname{div}\mathbf{D} = 4\pi q\delta(\mathbf{r}), \quad \mathbf{D}_{\mathbf{k}} = \varepsilon_l(0,k)\, i\,\mathbf{k}\varphi_{\mathbf{k}}$$

$$\varphi_{\mathbf{k}} = \frac{4\pi q}{k^2 + k_D^2}, \quad \varphi(r) = \frac{q}{r}\cdot e^{-rk_D}$$

static limit

$$\varepsilon_l(0,k) = 1 + \frac{k_D^2}{k^2}, \quad k_D^2 = \frac{4\pi e^2 N}{T}$$

Debye screening

Principal nonlocality

Force in the static limit, Pitaevskii, 2008

dielectrics

$$k_D \ll \frac{1}{a}$$

good metals

$$k_D \gg \frac{1}{a}$$

General case

$$\varepsilon_t(\omega,k)=\varepsilon_0(\omega)-\frac{\omega_p^2}{\omega(\omega+i\omega_\tau)}f_t(x) \qquad \varepsilon_l(\omega,k)=\varepsilon_0(\omega)-\frac{\omega_p^2}{\omega(\omega+i\omega_\tau)}f_l(x)$$

parameter of nonlocality $x=\dfrac{\omega+i\omega_\tau}{vk}$ x >> 1, local limit

$f_{t,l}(x)$ known for degenerate or nondegenerate plasmas

Room T:

$$\varepsilon_t=\varepsilon_l=\varepsilon(i\zeta_n)=\varepsilon_0(i\zeta_n)+\frac{\omega_p^2}{\zeta_n(\zeta_n+\omega_\tau)}, \quad n>0$$

$$\varepsilon_t(i\zeta,k)=\varepsilon_0+\frac{\omega_p^2}{\zeta\omega_\tau}, \quad \varepsilon_l(i\zeta,k)=\varepsilon_0+\frac{k_D^2}{k^2}, \quad \zeta\to 0$$

Low T: $\varepsilon_t(i\zeta_n,k)=\varepsilon_0(i\zeta_n)+\dfrac{\alpha\omega_p^2}{\zeta_n vk}, \quad \varepsilon_l(i\zeta_n,k)=\varepsilon_0(i\zeta_n)+\dfrac{k_D^2}{k^2}$ while x<<1

Landau damping

Low T: $\varepsilon_t(i\zeta_n,k)=\varepsilon_0(i\zeta_n)+\frac{\alpha\omega_p^2}{\zeta_n vk}, \quad \varepsilon_l(i\zeta_n,k)=\varepsilon_0(i\zeta_n)+\frac{k_D^2}{k^2}$ while x<<1

No dependence on ω_τ

At low T $\quad \omega_\tau(T) << vk \sim \frac{v}{a}$

Dissipation due to Landau damping in collisionless plasma

Electrons moving with $v \sim \frac{\omega}{k}$ see the field as stationary

The force applied to electrons produces work
(absorption of the field energy by collisionless plasma)

Entropy at T → 0

Landau damping changes behavior of free energy

$$\Delta\mathcal{F} = \frac{kT\,A^2}{8\pi\,a^2}\,(0.0146 - 0.0041A)\,, \qquad A = \left(\frac{3\pi^2}{2}\,\frac{c}{v_F}\,\frac{\omega_p^2}{\omega_c^2}\,\frac{kT}{\hbar\omega_c}\right)^{1/3} \ll 1$$

$$A \sim aT^{1/3}$$

$$\Delta F = C_1 T^{5/3} - C_2 T^2 a, \quad C_{1,2} > 0$$

does not depend on a

has no relation to the Casimir force

Entropy: $$S = -\frac{\partial \Delta F}{\partial T} = -\frac{5}{3} C_1 T^{2/3} + 2C_2 T a$$